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Spherical wave attenuation under multiple energy source in viscous fluid-saturated elastic porous media

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ABSTRACT

In viscous fluid-saturated elastic porous media, the spherical dilatational wave displacement equations are developed on Biot's theory and the spherical shear wave displacement equations are developed based on a modified wave frame. Analytical forms of the solutions are introduced by considering the wave magnitude dispersion due to the viscosity of Newtonian fluids. The attenuation of dilatational wave and shear wave are separately studied. Due to the high attenuation of the shear wave, the superposition characteristic is only considered for dilatational waves under the multiple energy source models. Many advantages of the multiple sources over the single source have been theoretically or numerically demonstrated, such as multiple energy sources can produce a larger relative displacement than single source, the wave direction is controllable and the magnitude of the relative displacement can be adjusted as required under the multiple source.

1. Introduction

Seismic waves generated from earthquakes and mechanical vibration could alter oil production by the vibrations propagating into the reservoir as elastic waves give rise to numerous effects on the fluids flow in porous rocks. Therefore, the increase of oil production with vibration stimulation ranged from 10% to 65% (Simkin and Surguchev, 1991; Cook and Sheppard, 1989; Kouznetsov et al., 1998; Baviere, 2007). Though numerous investigations were carried out and a considerably large number of theoretical research results have been reported in attempting to understand the principle of enhanced oil recovery (EOR) with vibration and seismic stimulation (Huh, 2006; Igor and Beresnev, 1994; Pujol, 2003; Roberts et al., 2003; Serdyukov and Kurlenya, 2007; Steven et al., 2008), a thorough comprehension of the effects of vibration and seismic excitations on EOR processes is far beyond being reached. The vibrations propagating into the reservoir as elastic waves give rise to numerous effects on the fluids flow in porous rocks. Furthermore, the wave motion needed for generating the excitation desired is not comprehended both at theoretical and practical levels. This limits the application of the seismic and vibration stimulation technique to be developed for industrial applications.

For understanding the relative motion between the fluid and solid, Wang et al., (2007, 2009) developed the wave governing equations based on Biot's theory in terms of the displacements in cylindrical coordinates. The relative motion between the fluid and solid of a porous medium is investigated by the proposed relative displacement. For both the models studied with or without fluid viscosity, the wave sources considered in Wang's works are taken as straight lines which can be merely expanding in a two dimensional plane and are insufficient to disclose the spatial wave propagation characteristics in reality. Han and Dai (2011, 2013) developed the non-viscous displacement equations in spherical coordinates for considering the wave propagated from point energy sources, whose dimensions are negligible in comparison with the wavelength of the waves that the point source assumption is more acceptable and rational than other types of wave source in practice. Although Han and Dai (2012) tried to set up the viscous fluid model in spherical coordinates, the shear wave characteristics were not considered that the research was incomplete.

In this research, to analytically describe the full wave behavior excited by multiple point energy sources in a porous medium with viscous fluid, the dilatational wave displacement function is developed in spherical coordinates based on Biot's Theory. The wave velocity depends on the material physical properties such as the saturation of pore and permeability, the selection of parameters has a strong effect on the quality of the propagation of both shear and compressional wave (Bala and Cichy, 2007; Castagna et al., 1985).

In addition, a modified shear wave model based on Sahay's (2008) theory is developed for better describing the dynamic behavior of shear waves in viscous fluid. It has been found out the slow shear wave vanished rapidly close to the wave source in the unbound porous

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medium and the fast shear wave is with high attenuation, which reaches the same conclusion as Biot. Therefore, due to the high attenuation of the shear waves, the superposition characteristic under multiple sources are considered for dilatational waves under the multiple energy source models. Many advantages of the multiple sources over the single source have been demonstrated, such as multiple energy sources can produce a larger relative displacement than single source. Other advantages of multiple energy sources over a single one are theoretically demonstrated in wave direction control and magnitude adjustment. Because of the viscosity involved, the frequency becomes critical in affecting the wave magnitude under given multiple sources and this effect has been demonstrated as well.

2. Governing equation for dilatational and shear wave

This section focuses on the development of the dilatational and shear wave displacement differential equations for an elastic solid containing a viscous fluid. The dilatational wave equations based on Biot's theory (Biot 1956a, b; Berryman, 1985; Gurevich, 1999; Wang et al., 2007, 2009; White, 1975), which allows the further measurement of the relative displacement between the solid and fluid. The shear wave equations are following the analysis of a modified stress-strain relations in a fluid-containing, porous elastic solid proposed by Sahay.

In Biot's theory, the wave equations in the low frequency range are derived based on the following assumptions: the relative motion of the fluid in pores is laminar viscous and incompressible; the pore size of the material is geometrically small in comparison with that of the unit solid-fluid element; the flow is through a constant circular cross-section that is substantially longer than its diameter, which requires the wavelength of the wave travelling in the porous medium to be much larger than that of the unit element itself. Under these assumptions, the governing equations of wave propagation with friction have been shown to be (Biot, 1956a, b):

$$N\nabla^2 u + [(A+N)e + Q\varepsilon] = \frac{\partial^2}{\partial t^2}(\rho_{12}u + \rho_{22}u) + b\frac{d}{dt}(u - U) \tag{1a}$$

$$\nabla[Qe + R\varepsilon] = \frac{\partial^2}{\partial t^2}(\rho_{12}u + \rho_{22}u) - b\frac{d}{dt}(u - U) \tag{1b}$$

where u and U are the displacement vectors of the solid and fluid, respectively, and e and ε are the volume strains of the same solid and fluid, respectively. The coefficients ρ_{11}, ρ_{12} and ρ_{22} are density coefficients which take into account the fact that the relative fluid flow through the pores is not uniform. These parameters can be expressed as $\rho_{11} = (1-\phi)\rho_s$ $\rho_{22} = \phi\rho_f$, and $\rho_{12} = -(\alpha - 1)\phi\rho_f$, where ρ_s and ρ_f are the mass densities of the solid and fluid, respectively, ϕ is the porosity of the porous medium.

$\alpha = \frac{(1+\phi^{-1})}{2}$. $\boldsymbol{A}$ corresponds to the familiar Lame coefficients in the theory of elasticity, where $\boldsymbol{N}$ represents the shear modulus of the material. The coefficient $\boldsymbol{R}$ is a measurement of the pressure on the fluid required to force a certain volume into the aggregate while the total volume remains constant. Q describes the coupling between the volume change of the solid and that of the fluid. The coefficient b is related to Darcy's coefficient of permeability k by $b = \frac{\mu\phi^2}{k}$. By solving the governing equations in Equations (1a) and (1b), Biot presented the wave expressions in the form of volume strain. However, these expressions cannot be directly used to quantify the displacement field in a three-dimensional domain. To describe the displacement field, the equations for the propagating waves need to be revised so that the displacements of the fluid and solid can be expressed separately and quantified by the governing equation. By applying the Helmholtz decomposition to the displacement vectors of the solid and fluid of a porous medium, the general form of the governing equations can be derived. Specifically, applying Helmholtz decomposition to the displacement vectors of the solid and fluid of a porous medium, the displacement vectors of the solid and fluid can be given by

$$u = \nabla\varphi_s + \nabla \times \psi_s \tag{2a}$$

$$U = \nabla\varphi_f + \nabla \times \psi_f \tag{2b}$$

where φ_s and φ_f are scalar potentials of the solid and fluid, whereas ψ_s and ψ_f are vector potentials for the displacements of solid and fluid, respectively. ψ_s and ψ_f also satisfy the following conditions $\nabla\cdot\psi_s = 0$ and $\nabla\cdot\psi_f = 0$.

For dilatational wave, also known as compressional wave or P wave, the displacement is caused by the scalar potentials without rotation, which implies that $\nabla \times (\nabla\varphi_s) = 0$ and $\nabla \times (\nabla\varphi_f) = 0$. For shear wave, also known as rotational wave or S wave, the displacement is due to vector potentials, such that $\nabla\cdot(\nabla \times \psi_s) = 0$ and $\nabla\cdot(\nabla \times \psi_f) = 0$. Substituting equation (2a) and (2b) into equations (1a) and (1b), and rearranging the terms according to the scalar and vector potentials, two sets of equations can be obtained corresponding to the scalar and vector potentials of the fluid and solid.

With the above considerations, the expressions for compressive wave can be given as follows

$$\nabla^2(P\varphi_s + Q\varphi_f) = \frac{\partial^2}{\partial t^2}(\rho_{11}\varphi_s + \rho_{12}\varphi_f) + b\frac{\partial}{\partial t}(\varphi_s - \varphi_f) \tag{3a}$$

$$\nabla^2(Q\varphi_s + R\varphi_f) = \frac{\partial^2}{\partial t^2}(\rho_{12}\varphi_s + \rho_{22}\varphi_f) - b\frac{\partial}{\partial t}(\varphi_s - \varphi_f) \tag{3b}$$

with the definition $P = A + 2N$. In the equations, the subscript s represents the displacement of solid; f represents the displacement of the fluid.

To develop wave equations in the form of displacements for P-wave, take the gradient operation to equations (3a) and (3b) such that

$$\nabla[\nabla^2(P\varphi_s + Q\varphi_f)] = \nabla\left[\frac{\partial^2}{\partial t^2}(\rho_{11}\varphi_s + \rho_{12}\varphi_f)\right] \tag{4a}$$

$$\nabla[\nabla^2(Q\varphi_s + R\varphi_f)] = \nabla\left[\frac{\partial^2}{\partial t^2}(\rho_{12}\varphi_s + \rho_{22}\varphi_f)\right] \tag{4b}$$

Let φ be a general scalar displacement potential and $\mathbf{u}$ a displacement for dilatational wave. The displacement vector $\mathbf{u}$, for dilatational wave, is merely related to the scalar potential. Hence, the displacement equation for dilatational wave can be given in general Laplacian operator form as the following

$$\nabla^2(Pu + QU) = \frac{\partial^2}{\partial t^2}(\rho_{11}u + \rho_{12}U) + b\frac{\partial}{\partial t}(u - U) \tag{5a}$$

$$\nabla^2(Qu + RU) = \frac{\partial^2}{\partial t^2}(\rho_{12}u + \rho_{22}U) - b\frac{\partial}{\partial t}(u - U) \tag{5b}$$

Under Biot's equations, for shear wave, the governing equations can be developed as

$$N\nabla^2\psi_s = \frac{\partial^2}{\partial t^2}(\rho_{11}\psi_s + \rho_{12}\psi_f) \tag{6a}$$

$$0 = \frac{\partial^2}{\partial t^2}(\rho_{12}\psi_s + \rho_{22}\psi_f) \tag{6b}$$

the magnitude of rotation denoted as $\boldsymbol{v}$ which is only related to the vector potential as $\mathbf{v} = \nabla \times \boldsymbol{\psi}$. To develop the displacement equations for shear wave, take the curl operation to the former equations (4a) and (4b) for shear wave such that

$$\nabla \times [N\nabla^2\psi_s)] = \nabla \times \left[\frac{\partial^2}{\partial t^2}\left(\rho_{11}\psi_s + \rho_{12}\psi_f\right)\right] \tag{7a}$$

$$0 = \nabla \times \left[\frac{\partial^2}{\partial t^2}(\rho_{12}\psi_s + \rho_{22}\psi_f)\right] \tag{7b}$$

The displacement equation for shear wave can then be given by

$$N\nabla^2 v = \frac{\partial^2}{\partial t^2}(\rho_{11}v + \rho_{12}V) + b\frac{\partial}{\partial t}(v - V) \tag{8a}$$