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Applicability of quasi-steady assumption during the numerical simulation of the start-up of weakly compressible Herschel-Bulkley fluids in pipelines

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ABSTRACT

In the numerical study of the start-up of viscoplastic fluids in pipelines, the quasi-steady assumption, *i.e.*, the assumption of a linear radial distribution of shear stress, is widely introduced into the governing equations to capture the radial position of the yield surface. However, few studies in the existing literature have examined the condition that the quasi-steady assumption is applicable. In the present work, we define a dimensionless number *Ut* for the start-up of weakly compressible Herschel-Bulkley fluids in pipelines. *Ut* increases with increasing pipeline aspect ratio, Reynolds number and flow index, and decreases with increasing fluid compressibility and Bingham number. Scale analysis and numerical studies show that the effect of these five parameters on the applicability of the quasi-steady assumption depends only on the value of *Ut*. The smaller the value of *Ut* is, the less likely the quasi-steady assumption would affect the simulation results. When $Ut < 0.065$, the difference in the time it takes for the outlet fluid starts to flow computed with and without the quasi-steady assumption is below 2%, and the difference in the outlet velocity during its recovery process computed with and without the quasi-steady assumption is below 1%. Moreover, the computed deviations slowly decrease as *Ut* decreases. Conversely, when $Ut > 0.065$, the quasi-steady assumption shortens the computed time it takes for the outlet fluid starts to flow and intensifies the computed transient processes of the velocities and pressures.

1. Introduction

The star-up of yield stress fluids is frequently encountered in pipeline operations (Bobert et al., 1997; Chang et al., 1999; de Oliveira et al., 2010; Ma et al., 2017; Negrão et al., 2011; Li et al., 2017; Li et al., 2018; Livescu, 2012; Lu et al., 2012; Paso, 2014; Sun et al., 2016; Taghavi et al., 2012). Numerical simulations are usually performed to guide a successful star-up (Ahmadpour et al., 2014; Chang et al., 1999; Davidson et al., 2004; de Oliveira et al., 2010, 2012; de Souza Mendes et al., 2012; Negrão et al., 2011; Sun et al., 2016; Vinay et al., 2006, 2007; Wachs et al., 2009). One major challenge when conducting such simulations is to capture the radial position of the yield surface (Ahmadpour et al., 2014; Vinay et al., 2006, 2007; Wachs et al., 2009). In order for yield stress fluids to flow, a shear stress larger than the yield stress needs to be imposed. As such, the constitutive equation of yield stress fluids is often a piecewise function which is non-differentiable at the yield point. A constitutive equation in such mathematical form is difficult solve simultaneously with the momentum equation.

Two approaches have been proposed in the literature to address the issue of non-differentiable constitutive equations. One approach involves the application of Lagrange multipliers technique and the augmented Lagrangian/Uzawa method, such as those applied in the work of Vinay et al. (2006, 2007) and Wachs et al. (2009). This approach transforms the governing equations (including the continuity equation, the momentum equation and the constitutive equation) into a problem that finds the minimal solution to an augmented Lagrangian function, and uses the Uzawalike algorithm to solve the equivalent saddle-point problem. This approach considers the real constitutive equation of the yield stress fluid and captures the exact radial position of the yield surface. Using this approach, Vinay and Wachs have successively established 2D (both the continuity equation and the momentum equation are written with the axial velocity) (Vinay et al., 2006), 1D (both the continuity equation and the momentum equation are written with the mean axial velocity) (Vinay et al., 2007) and 1.5D (the continuity equation is written with the mean axial velocity and the momentum equation is written with the axial velocity) (Wachs et al., 2009) numerical algorithms. In particular, the 1.5D model is a compromise between a fully 2D model and a fully 1D model, and yields simulation results as accurate as a fully 2D model while keeping the computational cost much more reasonable. They then studied the

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Nomenclature

v	axial velocity [m/s]
V	mean axial velocity [m/s]
L	pipeline length [m]
R	pipeline radius [m]
P	pressure [Pa]
ΔP	total pressure drop between the pipeline inlet and outlet [Pa]
t	time [s]
V_0	reference velocity [m/s]
Bn	Bingham number, $Bn = L\tau_y/\Delta PR$
Re	Reynolds number, $Re = \rho_0 V_0 R/\eta_0$
K	consistency for Herschel-Bulkley plastic [Pa·s^n]
n	flow index
Ut	order of magnitude of the transient term in the dimensionless momentum equation (Eq. (17))
RT	relative deviation of the time at which the fluid at the pipeline outlet starts to flow computed with the quasi-steady assumption (Eq. (19))
RP	average relative deviation of the outlet velocity during its recovery process computed with the quasi-steady assumption (Eq. (20))
N	number of nodes

Greek symbols

$\dot{\gamma}$	shear rate [s^{-1}]
τ	shear stress [Pa]
τ_y	yield stress for Herschel-Bulkley plastic [Pa]
ρ	oil density [kg/m^3]
ρ_0	oil density at the pipeline outlet [kg/m^3]
τ_w	shear stress at the pipeline wall [Pa]
α	compressibility of the fluid [Pa^{-1}]
δ	aspect ratio of the pipeline, $\delta = R/L$
η_0	reference viscosity [Pa·s]
η_{ap}	apparent viscosity [Pa·s]

Subscripts

z	axial direction
r	radial direction
in	pipeline inlet
out	pipeline outlet

Superscripts

*	dimensionless variable
k	time step
i	loop count
j	radial position of finite volume

effects of fluid compressibility, pipeline aspect ratio and Reynolds number on the start-up of weakly compressible viscoplastic fluids in pipelines. In a separate study, Ahmadpour et al. (2014) combined this approach with Houska model to study the effect of plastic viscosity on the start-up of viscoplastic/thixotropic fluid.

Another widely-used approach to address the non-differentiable constitutive equation is to assume that the shear stress has a linear radial distribution. This approach is also referred as the quasi-steady assumption. (Chang et al., 1999; Davidson et al., 2004; de Oliveira et al., 2010, 2012; de Souza Mendes et al., 2012; Negrão et al., 2011; Sun et al., 2016). Using the Fanning friction factor derived under the steady state condition, a 1D numerical algorithm can be established to study the start-up flow (Chang et al., 1999; Davidson et al., 2004; de Oliveira et al., 2010, 2012). The 1D model can provide the time and axial variations of the mean axial velocities and pressures without knowing the radial position of the yield surface, but it cannot predict the radial distribution of the axial velocity. Given this limitation, Negrão et al. (2011) combined the quasi-steady assumption and a search algorithm to determine the axial distribution of the pipeline wall shear stress, and proposed a numerical algorithm for the start-up flow of drilling fluids in pipelines. The numerical algorithm is similar to the 1.5D numerical algorithm established by Wachs et al. (2009), and can deliver the time and axial evolution of the mean axial velocities and the pressures as well as the radial distribution of the axial velocity and structural parameter. In recent work, de Oliveira and Negrão (2015) adopted the quasi-steady assumption to study the effect of fluid compressibility on the start-up of elasto-viscoplastic fluid, and Sun et al. (2016) studied the influence of rheological changes of emulsion gel on pipeline restart process based on the quasi-steady assumption.

Compared to Lagrange multipliers technique and augmented Lagrangian/Uzawa method, the approach of quasi-steady assumption is efficient in simplifying the momentum equation and numerical algorithm. Nevertheless, the restart of pipeline is a transient process, and as an approximation method, the quasi-steady assumption inevitably contains ‘application condition’. In other words, the assumption will influence the numerical simulation results to a varying extent. Although it has been applied widely, there are few reports about the application condition of the quasi-steady assumption. Only de Oliveira et al. (2010) compared the computed results of two studied cases with those of Vinay et al. (2007) when studying the start-up flow of weakly compressible Bingham fluids. The results show that the computed results of one case are completely consistent with those obtained by Vinay et al. (2007), whereas the computed results of the other case, in which the fluid of interest is less viscous and more weakly compressible, are different. The authors attributed these discrepancies to the coarse mesh used in the work of Vinay et al.

The present work addresses the start-up flow of weakly compressible Herschel-Bulkley fluids in pipelines. First, the effect of introducing the quasi-steady assumption into the governing equations was theoretically analyzed using scale analysis, and the differences in the simulation results numerically computed with and without the quasi-steady assumption were analyzed by changing the values of the pipeline aspect ratio, the Reynolds number, the flow compressibility, the Bingham number and the flow index. Finally, the critical condition when the quasi-steady assumption is applicable was established according to the theoretical analysis and numerical analysis.

2. Start-up flow models

2.1. Physical model

Various physical models have been proposed to describe the start-up of structured fluids in pipelines (Ahmadpour et al., 2014; Chang et al., 1999; de Oliveira et al., 2010, 2012; Davidson et al., 2004; de Souza Mendes et al., 2012; Huilgol, 2015; Negrão et al., 2011; Sun et al., 2016; Vinay et al., 2006, 2007; Wachs et al., 2009). In the present work, we adopted the one that is widely accepted by most studies. (Ahmadpour et al., 2014; de Oliveira et al., 2010, 2012; de Souza Mendes et al., 2012; Sun et al., 2016; Vinay et al., 2006, 2007; Wachs et al., 2009). As shown in Fig. 1, the pipeline is initially filled with a weakly compressible homogeneous Herschel-Bulkley fluid. A constant pressure is applied at the pipeline inlet, in other words, the constant pressure restart way is chosen. The Herschel-Bulkley fluid in the pipeline yield and flow gradually from upstream to downstream until it