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A stability-improved efficient deconvolution algorithm based on B-splines by appending a nonlinear regularization

Wenchao Liu[a,b,*], Yuewu Liu[b], Weiyao Zhu[a], Hedong Sun[c]

[a] School of Civil and Resource Engineering, University of Science and Technology Beijing, Beijing 100083, PR China
[b] Institute of Mechanics, Chinese Academy of Sciences, Beijing 100190, PR China
[c] Department of Gas Field Development, PetroChina Research Institute of Petroleum Exploration and Development, Langfang, Hebei 065007, PR China

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ABSTRACT

Previous deconvolution algorithms based on B-splines are much easier to be understood and programmed for academic researchers and engineers. However, due to the use of a linear regularization, their stability is weaker than that of the commonly used von Schroeter et al.'s deconvolution algorithm in which a nonlinear regularization is used; the linear regularization can make the deconvolution algorithms less tolerant to data errors. Good stability for the deconvolution algorithms is very important in order to make deconvolution as a viable tool for well-test analysis. In the paper, in order to improve the stability of the deconvolution algorithms based on B-splines, a nonlinear regularization by minimizing the curvature of pressure derivative response, as used in von Schroeter et al.'s algorithm, is appended instead of the linear regularization. And the corresponding nonlinear regularization equations are appropriately deduced. In particular, the improved algorithm is based on the Duhamel principle directly, and the complex transformation by the nonlinear *z* function, as used in von Schroeter et al.'s algorithm, is avoided; it does simplify the whole deconvolution process; moreover, the sensitivity matrix of an involved basic linear system from the measured pressure and rate data can also be solved directly by the piecewise analytical integration method, which can largely improve the deconvolution computation speed. Ultimately, in combination with the nonlinear regularization equations, a nonlinear least-squares problem is formulated for the stability-improved deconvolution algorithm based on B-splines. Besides, a constraint condition for tuning the parameter values of the B-spline base and an involved smooth factor is presented for restricting the nonlinear regularization process. Through a simulated case study, it is found that the nonlinear least-squares problem can be solved stably by the advanced Powell's Dog Leg method due to its great convergence ability and numerical stability; and the solution accuracy is also verified. Then the effects of the two parameters on the type curves of the deconvolution results are analyzed. And the effect of the error in the initial formation pressure on the type curves of the deconvolution results is also analyzed. Then a statement on how to perform the nonlinear regularization is presented specifically.

Furthermore, through the study on two simulated cases with added data errors and an actual case, it is demonstrated that when the nonlinear regularization is appended, the stability of the deconvolution algorithm based on B-splines can be largely improved for mitigating the effect of data errors; besides, the stability-improved algorithm based on B-splines even exhibits higher stability than von Schroeter et al.'s algorithm that takes the same nonlinear regularization method, and the reason can be attributed to the superior properties of the representation of the wellbore pressure derivative (to be deconvolved) by B-spline functions in the numerical stability of computations and the inherent smoothness. Through the test of some simulated cases, it is also concluded that the stability-improved algorithm based on B-splines by appending the nonlinear regularization still has a high-level computation speed, which is nearly twenty times more than that of von Schroeter et al.'s algorithm. It can be attributed to the more undetermined coefficients and the computational complexity resulted from the *z*-function transformation in the formulation of von Schroeter et al.'s algorithm.

* Corresponding author. School of Civil and Resource Engineering, University of Science and Technology Beijing, Beijing 100083, PR China.
E-mail address: wcliu_2008@126.com (W. Liu).

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1. Introduction

The deconvolution based on Duhamel principle has been widely applied in the well testing technology in reservoir engineering. The inverse problem can provide the equivalent constant unit production rate pressure response of the well in a reservoir system that is affected by the variable production rates for the entire duration of the production history. The relevant deconvolution algorithms have attracted big attentions over the past forty years (Liu et al., 2017). Due to the commonly existent errors of wellbore pressure and production rate data in the fields, the deconvolution computation is always ill-conditioned inherently (Çınar et al., 2006). As far as we know, although many deconvolution algorithms have been proposed, just several ones appear to exhibit the stability of data error tolerance; they are proposed by von Schroeter et al. (von Schroeter et al., 2002; von Schroeter et al., 2004), Levitan et al. (Levitan, 2005; Levitan et al., 2006) and Ilk et al. (Ilk, 2005; Ilk et al., 2005; Liu et al., 2017), respectively. Here, these aforementioned different deconvolution algorithms will be introduced in details. In addition, it is worth to mention that recently Ahmadi et al. (2017) present a new robust deconvolution algorithm with the minimum user interference, which combines the conveniences of deconvolution in Laplace domain with a new approach to transform the sampled data from time domain to Laplace domain without extrapolating the data beyond the sampling interval; Ahmadi et al.'s algorithm overcomes the limitations of the requirement that the piecewise functions for the sampled data representation should be defined in the complex plane for the application of deconvolution in Laplace domain (Al-Ajmi et al., 2008).

It is well known that the Duhamel principle (Çınar et al., 2006) is as follows:

$$p_{\mathrm{ini}} - p = \int_0^t q(t-\tau) p_{\mathrm{u}}^{'}(\tau)\mathrm{d}\tau \tag{1}$$

where t is the time; τ is a variable for the integral; q is the measured variable rate; p is the measured wellbore pressure corresponding to the variable rate; p_{u} is the wellbore pressure drop corresponding to the constant unit rate; p_{ini} is the initial formation pressure. The aim of these deconvolution algorithms is to obtain p_{u} when the data of q and p are both given.

In order to make sure the positivity of $\mathrm{d}p_{\mathrm{u}}/\mathrm{dln}(t)$ for the relevant plotting of type curves, z function is defined in von Schroeter et al.'s deconvolution algorithm (von Schroeter et al., 2002; von Schroeter et al., 2004), as follows:

$$z = \ln\left[\frac{\mathrm{d}p_{\mathrm{u}}(t)}{\mathrm{dln}(t)}\right] \tag{2}$$

Then Eq. (1) can be equivalently transformed as follows:

$$p_{\mathrm{ini}} - p = \int_{-\infty}^{\ln t} q(t-\mathrm{e}^{\tau})\mathrm{e}^{z(\tau)}\mathrm{d}\tau \tag{3}$$

Then the aim turns to the solution of z. von Schroeter et al.'s deconvolution algorithm (von Schroeter et al., 2002; von Schroeter et al., 2004) accounts for the fitting errors for both the measured pressure data and rate data; in order to improve the smoothness of the solution of z when data errors exist, minimization of the curvature of z function is appended as a nonlinear regularization method. As a result, a total nonlinear least-squares problem is formulated. As for Levitan et al.'s deconvolution algorithm (Levitan, 2005; Levitan et al., 2006), their ideas are also from von Schroeter et al.'s deconvolution algorithm. They are both based on the same concept of minimizing a nonlinear weighted least-square objective function, involving the sum of three mismatch terms including the pressure, the rate and the curvature, for reconstructing the deconvolved pressure drop and its logarithmic derivative (Liu et al., 2017). The difference of the two algorithms mainly lies in the aspects of model assumption and the specific definition of objective functions. Due to the use of nonlinear regularization i. e. the minimization of the curvature instead of the pressure derivative (Ilk, 2005; Ilk et al., 2005), von Schroeter et al.'s deconvolution algorithm can exhibit relatively higher stability when data errors exist (Çınar et al., 2006).

Another different deconvolution algorithm based on B-splines is proposed by Ilk et al. first (Ilk, 2005; Ilk et al., 2005). The algorithm is based on Eq. (1) directly, and the transformation of Eq. (1) by the nonlinear z function is avoided; a weighted summation of second-order B-splines is adopted to reconstruct $p_{\mathrm{u}}^{'}$; and a linear regularization method is adopted to overcome the effect of data errors, which can make the logarithmic derivative of p_{u} differ slightly between the B-spline knot and the middle location between knots (Ilk, 2005; Ilk et al., 2005). In combination with Laplace transform and numerical Laplace inversion, the formulated linear least-squares problem can be solved. What's more, Ilk et al.'s algorithm is further improved by Liu et al. (2017) through a technique of piecewise analytical integration for calculating the involved sensitivity matrix (Liu et al., 2017) in the real time space instead of the Laplace space; then the success of the deconvolution computation based on B-splines can be guaranteed, and the improved deconvolution algorithm exhibits big advantage in the fast computational speed due to the use of the analytical solution method.

Good stability of deconvolution algorithms is very necessary in order to make deconvolution as a viable tool for well-test analysis; and stability improvement is also the main difficulty in the development of deconvolution algorithms. Çınar et al. (2006) have ever conducted a comparative study on these deconvolution algorithms mentioned above: Significantly, it is found that the weaker linear regularization method applied in the deconvolution algorithms based on B-splines (including Ilk et al.'s deconvolution algorithm (Ilk, 2005; Ilk et al., 2005) and its improved version by Liu et al. (2017)) can make the algorithms less tolerant to data errors; in contrast, von Schroeter et al.'s deconvolution algorithm shows relatively higher stability by using the nonlinear regularization method, and the deconvolution algorithm has been implemented into Saphir as the pressure transient analysis module of KAPPA software due to its good stability. However, in von Schroeter et al.'s deconvolution algorithm, the transformed deconvolution equation of Duhamel principle i. e. Eq. (3) is used, which makes the computation process become very complicated; in contrast, the deconvolution equation of Duhamel principle i. e. Eq. (1) is directly used in the algorithms based on the B-splines, and the involved sensitivity matrix can also be solved directly by the piecewise analytical integration method, which can largely improve the computation speed; and its computation procedures are much easier to be understood and programmed for academic researchers and engineers. What's more, representation of the unknown function by B-spline functions, which are piecewise defined polynomial functions, has superior properties such as local effects of coefficients, numerical stability of computations and inherent smoothness (Jauch et al., 2017) in comparison with representation of the unknown function by piecewise linear approximations as used in von Schroeter et al.'s algorithm. Therefore, in view of the above-mentioned facts, it is very necessary to further improve the stability of the algorithms based on B-splines. For the purpose, an idea can be presented naturally that the nonlinear regularization method in von Schroeter et al.'s algorithm may be applied into the algorithms based on B-splines.

In this paper, based on the improved version of Ilk et al.'s deconvolution algorithm (Liu et al., 2017), the nonlinear regularization method, as used in von Schroeter et al.'s deconvolution algorithm, is appended so as to largely improve the stability of the algorithms based on B-splines. It can make the deconvolution algorithm based on B-splines to be more acceptable and more advanced for its applications in the well testing technology. The schematics for the stability improvement of the deconvolution algorithm based on B-splines are shown in Fig. 1. From Fig. 1, it can be seen that the stability-improved algorithm can inherit good "genes" including the representation of $p_{\mathrm{u}}^{'}$ by B-splines (Liu et al., 2017), no complicated z-function transformation of the convolution equation (Liu et al., 2017), the fast analytical solution method for calculating the elements of sensitivity matrix (Liu et al., 2017) and the nonlinear