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Effect of wellhead tension on buckling load of tubular strings in vertical wells

Qiang Zhang [a,*], Bao Jiang [a], Wenjun Huang [b,c,**], Wei Cui [a], Jubao Liu [a]

[a] College of Mechanical Science and Engineering, Northeast Petroleum University, Daqing 163318, PR China
[b] School of Aerospace Engineering, AML, Tsinghua University, Beijing 100084, China
[c] School of MOE Key Laboratory of Petroleum Engineering, China University of Petroleum, Beijing 102249, China

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ABSTRACT

The geometrical and contact nonlinearities in tubular buckling problem lead to convergence difficulty in calculation. To solve this problem, we present a slow dynamic method and its solution strategies for the nonlinear static buckling analysis based on the implicit finite element method. For different length and boundary conditions, we calculate the length of each section of the helical buckling configuration. To measure the pitch of helical buckling, we introduce two methods. The first method is to use the spiral angle between the bottom and top contact points to measure the pitch, and the second method is to use the spiral angle of the continuous contact section to measure the pitch. For the first method, the string has three types of buckling configurations for different boundary conditions without the tensile section. With the tensile section, the helical buckling configuration is composed of the bottom compressed section, the middle helically buckled section, the top compressed section and the tensile section for the hinged or clamped boundary at both ends. For the second method, the buckling configuration consists of four non-contact sections, one continuous contact section without the tensile section. A tension section is added to the buckling configuration for the influence of the tension section. The critical load decreases gradually and tends to the minimum with the effect of the tension section. Since the critical load of the second methods is greater than the value of the first one, it is recommended that the former method be adopted in engineering applications.

1. Introduction

Tubular strings constrained by wellbores are subjected to the action of a compressive force in the bottom due to their own weight. When the bottom axial force exceeds a certain value, its elastic stability is lost and enters into buckling state. For instance, drill string buckling could change the bit direction, increase the lateral force and friction force, and make drill strings lock-up, even fatigue failure.

The analysis of critical buckling load of tubular string in vertical wellbores is one of the important problems. Lubinski (1950) firstly studied the stability of drill strings in vertical wellbores. He deduced the bending equation of drill strings in two-dimensional plane and its series solutions with the beam-column model. For the hinged drill string with a dimensionless length of 8, Lubinski gave the calculation formula of initial critical load of drill strings in vertical plane:

$$F_{\text{sin}} = 1.94\cdot\sqrt[3]{EIq^2}. \tag{1}$$

In fact, the above critical load is not affected by the wellbore constraints, similar to the critical load of compressed bars applied by compressive force investigated by Timoshenko and Gere, (1961).

When the tubular string occurs the initial buckling above, the string is in contact with the wellbore with the increase of the compressive force on the bottom. The sinusoidal and helical buckling would successively occur. Lubinski et al. (1962) firstly put forward the concept of helical buckling and proposed the calculation method for this buckling. They assumed that the buckling deformation was a three-dimensional helix and deduced the pitch-force relationship with the energy method

$$F_{\text{hel}} = 8\pi^2 EI/p^2. \tag{2}$$

Moreover, obviously the helical buckling configuration in the compression section is a helix with variable pitches in fact, and the pitch

* Corresponding author.
** Corresponding author. School of Aerospace Engineering, AML, Tsinghua University, Beijing 100084, China.
E-mail addresses: tical2012@163.com (Q. Zhang), huangwenjun1986@126.com (W. Huang).

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become larger when closer to the neutral point.

Mitchell (1988) also studied tubular string helical buckling and proved that the helical buckling model presented by Lubinski was just an approximate result. Mitchell's results indicated that the Lubinski pitch-force relationship became invalid to depict the tubular buckling behavior near the neutral point because the tubular string might be not in contact with the wellbore.

Under the assumption of variable pitches, Kwon (1988) analyzed the helical buckling of vertical tubular string with weight. Calculation formula of pitches was achieved through solving the general fourth-order nonlinear differential equation using virtual work principle. It should be noted that in Kwon equation, if $z=0$, the obtained calculation formula of pitches was in line with formula (2) and could be expressed as

$$p = 4.29 \cdot \sqrt[3]{EI/q}. \tag{3}$$

According to Kwon equation, the first pitch on the top showed large differences when the length of compression section were different. For example, the dimensional compression section lengths of 4.29, 15.50, 21.93 and 29.35 corresponded to the 1th, 5th, 8th and 12th non-uniform pitch, respectively. Then the dimensional length of the first pitch on the top were 4.29, 4.79, 5.09 and 5.40, respectively, signifying Kwon formula was non-universal.

Zhang (1989) also investigated the helical buckling of tubular string with weight and deduced the formula of variable pitches with energy method:

$$p_i = \sqrt[3]{9i^2\pi^2 EI/q} - \sqrt[3]{9(i-1)^2\pi^2 EI/q}, \tag{4}$$

where i is the ith full helix counted from top to bottom. If $i=1$, the first pitch at the top is

$$p = 4.46 \cdot \sqrt[3]{EI/q}. \tag{5}$$

In addition, Wu (1992), Gao (1996, 2006), Mitchell (2002, 2012), Lukasiewicz and Knight, (2006), Thompson et al., (2012, Thompson and Heijden, 2013), Gulyayev et al., (2014), Sun and Lukasiewicz, (2006, Sun et al., 2015) and Yue et al., (2017) etc. also investigated the helical buckling of tubular string using different theoretical or experimental methods. Hajianmaleki and Daily, (2014) researched the helical buckling of tubular string with weight in vertical wellbores with the explicit finite-element method by ABAQUS software. Table 1 displays the dimensionless pitches of helical buckling in vertical wellbores reported in some literature, the maximum value is 35.57% higher than the minimum.

The helical buckling formulas of tubular string mentioned above were obtained under the assumptions of constant or variable pitches, boundary conditions were universally ignored. However, in addition to the wellbore constraint, the vertical well string has boundary constraints on both ends. Furthermore, the current researches mainly focused on the buckling below the neutral point. The effect of tension section above the neutral point on the buckling was not taken into account. In particular, for the tubular string suspended in vertical wellbores, its top and bottom are subjected to the action of a tensile force and a compressive force respectively under its own weight, which results in the buckling problem more complex.

Although researchers realized that tubular string helical buckling cannot happen until a full pitch was formed, the calculated pitch lengths in helical section were obviously different as shown in Table 1. The critical helical buckling load was defined as the weight on the bottom of the continuous contact section. That is to say, the dimensionless pitch in helix section of tubular string was the dimensionless critical helical buckling load. For example, Gao (2006) calculated the pitch in helix section and the pitch was $p = 5.62 \cdot \sqrt[3]{EI/q}$. The corresponding critical helical buckling load was $F_{\mathrm{hel}} = 5.62 \cdot \sqrt[3]{EIq^2}$. The effects of boundary conditions on both ends and the tubular string length on the helical buckling were not considered. Hence, this method was not suitable to calculate critical helical buckling load of the suspended tubular string.

Gao and Huang, (2015) looked forward to the research methods of helical buckling for suspended tubular string and put forward a research idea: the top suspended section was depicted by the beam-column model and the bottom continuous contact section was depicted by the buckling differential equation. In view of this research assumption, Huang et al., (2016) divided the string into four suspended sections and one continuous contact section. The buckling problem of the tubular string was transformed into a system of nonlinear equations by substituting relevant continuity, boundary and stability conditions. He solved these equations with iteration method and obtained the critical buckling loads.

For tubular string suspended in wellbore, the geometrical and contact nonlinearities in helical buckling problem lead to convergence difficulty in calculation. Therefore, we propose slow dynamic method and its solution strategies to analysis helical buckling of tubular string. We investigate the influence factors in the finite element algorithm by examples. Then we research the effects of suspended tension forces, tubular string lengths and boundary conditions on critical helical buckling load.

2. Mechanical model

In order to investigate the critical helical buckling load of the suspended tubular string in vertical wellbores, we establish a mechanical model as shown in Fig. 1. It is assumed that: (a) the tubular string and wellbore are characterized by constant cross section and the effect of tool joints etc. on buckling is neglected; (b) external loads include the suspended tension force on the top of the tubular string and its weight; (c) the tubular string is linear elastic; (d) the tubular string and wellbore has initial annular clearance, and the tubular string is concentric with the wellbore before deformation, as shown in Fig. 1 (a); (e) the friction force between the tubular string and wellbore's wall is ignored; (f) the final stable state of static helical buckling will be researched and the transient effect of lifting and lowering tubular string is neglected.

For example, the both ends are hinged in the mechanical model. The ends are subjected to lateral restraints. In addition, the suspended tension force and axial constraint are applied, respectively, on the top and bottom ends of the tubular string. Due to its own weight, the top of tubular string is subjected to the tensile force, expressed in $F_{\mathrm{H}} = \xi_{\mathrm{T}} \cdot \sqrt[3]{EIq^2}$. The reaction force of axial constraint is the compressive force on the bottom, expressed in $F_{\mathrm{B}} = \xi_{\mathrm{C}} \cdot \sqrt[3]{EIq^2}$. The suspended tensile force subjected on the top is small and the compressive force applied on the bottom is large, leading to the tubular string buckled, as shown in Fig. 1 (b).

The tubular string in vertical wellbores is flexible. When the tensile force on the top decreases, the axial compressive force on the bottom is increased, the stiffness in compressive section decreases. Then the tubular string buckles and loses its stability. Then lateral bending occurs, resulting in large lateral displacement and rotation. The relationship between compressive force and deformation of tubular string is no longer linear. It belongs to the geometrical nonlinear problem. During the lateral deformation, the tubular string is constrained by the wellbore. Contact

Table 1
Dimensionless pitches of helical buckling in vertical wellbores.

Author	Lubinski et al., (1962)	Kwon (1988)	Zhang (1989)	Wu (1992)	Gao (1996)	Gao (2006)	Hajianmaleki and Daily, (2014)
Dimensionless pitch ($\sqrt[3]{EI/q}$)	4.29	4.29–5.40	4.46	5.55	5.816	5.62	5.25
Remarks	*,#	**, #	**	**, #	Asymptotic solution	**, #	**, Finite element method

*Uniform pitch, ** Non-uniform pitch, # Energy method.