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Simplified modeling of YPL fluid flow through a concentric elliptical annular pipe

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ABSTRACT

Accurate prediction of the pressure gradient is the prerequisite for precise control of downhole pressure; such control can effectively avoid the complicated underground situations caused by improper downhole pressure control, such as well leakage, overflow and wall instability. The existing model of the annular pressure gradient in drilling process only considers a perfectly cylindrical wellbore; however, the real drilling hole easily becomes an elliptical wellbore under non-uniform in situ stress, leading to the relatively large error of the conventional model. Based on the theories of fluid mechanics, the analytical and simplified models of steady-state laminar flow in the concentric elliptical annular (elliptical wellbore) pipe are established. The analytical model is validated by the computational fluid dynamics (CFD) simulations of Fluent software. In addition, the simplified model was validated by analytical solutions. The primary conclusions are as follows. (1) When the flow rate is constant, the axial average and maximum velocities increases linearly and exponentially, respectively, with the increase of ε. The velocity profile of the analytical model is in good agreement with the velocity profile of the CFD simulations. (2) The CFD simulations are in perfect agreement with the analytical results, and the error of the pressure gradient is within $\pm 10\%$ error bands for pressure gradient. (3) The simplified model predictions are in good agreement with the analytical model results, and the error of most of the results is within $\pm 5\%$ bars. The model is convenient for use in predicting the pressure gradient accurately for yield power law (YPL) fluid flowing in an elliptical wellhole.

1. Introduction

Conventional pressure gradient prediction usually considers a perfectly cylindrical wellbore with concentric and eccentric annuli. A large number of studies have been conducted in a cylindrical wellbore with concentric and eccentric annuli. However, under the influence of non-uniform in situ stress and other factors, the wellbore formed by drilling is often not a perfectly cylindrical wellbore. The elliptical wellbore can be judged indirectly using the dual caliper logging curve. Fig. 1 shows the results of the dual caliper logging curve for the Daniudi Gas Field of PG3 and PG1 wells. The results show that the actual wellbore is an irregular cylindrical wellbore, with the ratio of the dual calipers being in the range of 1–1.2. To realize accurate prediction of the annular pressure drop or downhole pressure, it is of great significance to study the annular flow law of the elliptical wellbore.

Two general equations are used to predict the flow rate and the maximum velocity versus pressure drop relationships in the isothermal and laminar flow of non-Newtonian fluids in ducts of arbitrary cross section (contained elliptical ducts) (Kozicki et al., 1966). Moreover, two parametric constants (a and b) are introduced to predict the pressure drop for non-Newtonian fluid flow in elliptical ducts; the expressions for evaluation of a and b for this shape of cross-section are given by Eq. (1). Comparison of the predicted results with existing experimental data revealed good agreement for power law fluid flow through elliptical ducts.

$$\begin{cases} a+b=\dfrac{\pi^2}{8E_0^2}\left(\beta^2+1\right) \\ a=\dfrac{\pi^2}{32E_0^2}\left(\beta^2+1\right) \end{cases} \tag{1}$$

where E_0 is the elliptical integral of the second type, dimensionless. β is the ratio of the length of the minor axis to the length of the major axis, dimensionless.

Based on the methods previously reported by Boucher and Alves (1963), the correlation function of the shape factor (λ) was introduced to

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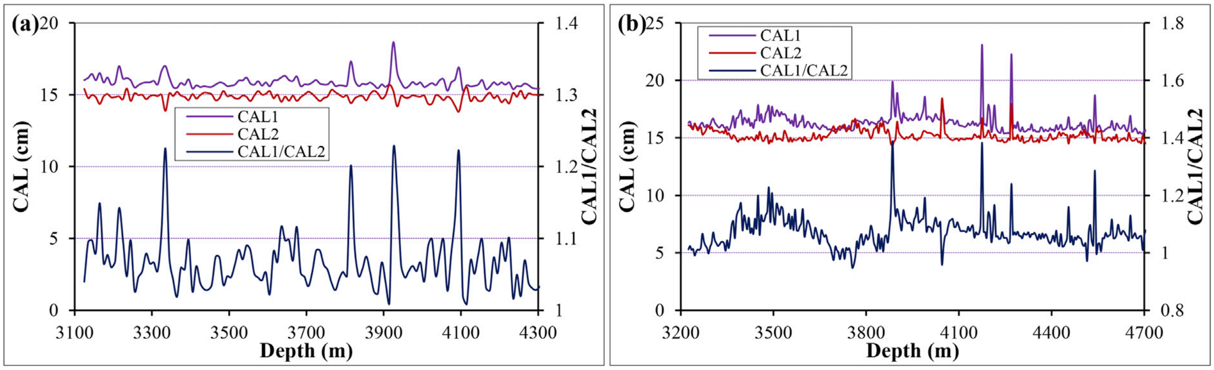

Fig. 1. Measurements of dual calipers for the Daniudi Gas Field: (a) PG3 well and (b) PG1 well.

predict pressure drop for Newtonian fluid flow in elliptical ducts (Miller, 2002). The expression for evaluation of λ for elliptical ducts is

$$\lambda = 19.75\left(1 - 0.31\beta + 0.12\beta^3\right) \tag{2}$$

A new generalized Reynolds number involves only one geometrical parameter (ξ), which can be easily determined theoretically or experimentally (Delplace and Leuliet, 1995). This Reynolds number was successfully compared with definitions found in the literature. Moreover, this generalized Reynolds number appears to be in good agreement with the experimental studies conducted in plate heat exchangers.

Fully developed laminar flow in noncircular ducts for non-Newtonian power law fluids was examined. Moreover, a simple model was developed using the effective hydraulic diameter (square root of cross-sectional area) to replace the hydraulic diameter; this model provides accuracy of better than $\pm 6\%$ for elliptical ducts.

The pressure drop of fully developed, laminar, incompressible flow in smooth mini- and micro-channels of arbitrary cross-section have been investigated (elliptical ducts) (Bahrami and Yovanovich, 2005). A compact approximate model was proposed that predicts the pressure drop for a wide variety of shapes. The proposed model was compared with analytical and numerical solutions for several shapes. In addition, the comparison of the model with experimental data collected by several researchers showed good agreement.

Analytical solutions were presented for laminar fully developed flow in the micro/mini-channels of hyper-elliptical cross-sections (Tamayol and Bahrami, 2009). The predicted results for the velocity distribution and pressure drop were successfully compared with existing analytical solutions, and experimental data were in good agreement with the prediction.

A narrow-slot model was established to predict the pressure gradient for YPL fluid flow through two types of elliptical cross-sectional ducts: a concentric elliptical annulus and an eccentric one (Alegria et al., 2012). The model was validated by comparing the results with the literature and with numerical results obtained from CFD. Very good agreement for the eccentric annular pipe was obtained from the comparison between the current analytical results and the numerical solutions from the literature.

CFD simulations were conducted to investigate non-Newtonian fluids flow through elliptical annulus (Dawood et al., 2015; Eid, 2011) and elliptical ducts (Maleki and Sadrhosseini, 2016) for laminar and turbulent flow states.

Many studies have been conducted to investigate the velocity and pressure gradient in elliptical ducts and annulus. However, the correlation models were established only for flowing in elliptical ducts. As is well-known, no simplified model for YPL fluid flow in the elliptical annulus is reported.

2. Analytical modeling

An analytical model (AM) has been developed to predict the pressure gradient for YPL fluids in an elliptical wellbore with concentric pipes based on the analytical solution of the equations of motion of axially laminar flow. The following assumptions are made to develop the model: (i) incompressible fluid; (ii) steady laminar and isothermal flow; (iii) the inner and outer walls are stationary; (iv) wall slippage effects are negligible; and (v) the wellbore is perfectly elliptical; (vi) the inner pipe is a closed-ended pipe.

From the geometric relationship (Fig. 2), the distance between the center and the outer walls at different θ can be determined by

$$h(\theta) = \sqrt{a^2\cos^2\theta + b^2\sin^2\theta} \tag{3}$$

where *h(θ)* is the distance between the center and the outer walls at different θ, m. θ is the wellbore azimuth angle, Deg. a is the major semi axis of the elliptical wellbore, m. b is the minor semi axis of the elliptical wellbore, m.

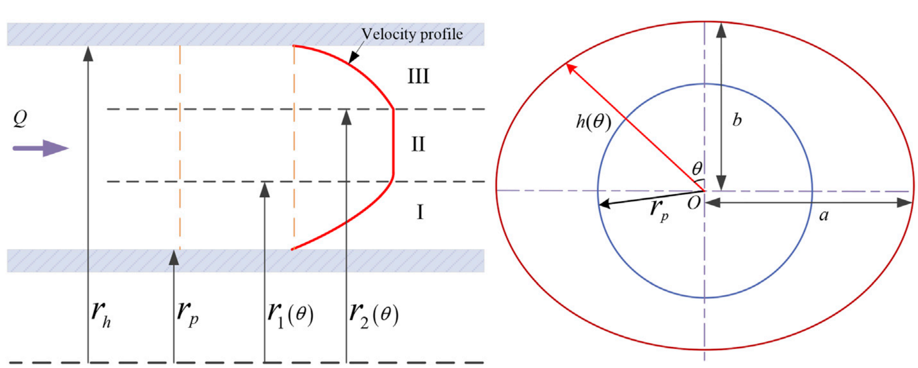

Fig. 2. Velocity profile of YPL fluid flow in elliptical annulus.