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# Simplified generalised drift velocity correlation for elongated bubbles in liquid in pipes

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ABSTRACT

Most of the existing drift velocity models have limitations, and sometimes low predictive capabilities, primarily because they are derived from experimental data which scarcely account for the combined effect of viscosity, surface tension and pipe inclination. Published data of drift velocity of elongated bubbles in pipes have been extracted from the open literature, and new data have been generated from Taylor bubble experiments conducted in a low pressure flow loop using nominal oil viscosities of 160cP and 1140cP in 0.099 m and 0.057 m internal diameter inclined pipes (1.0–7.5° from horizontal). These data have been processed and a simplified generalised drift velocity correlation established. The evaluation of some existing elongated bubble rise velocity has also been carried out. The prediction of the drift velocity of a single elongated gas bubble in liquid in pipes can sometimes be over-estimated by 20% or more, and sometimes be under-estimated by 20% or more. It is shown that the new proposed simplified generalised correlation has an improved predictive capability when used to estimate the drift velocity of a bubble in stagnant liquid in a pipe.

## 1. Introduction

In the production and transportation of oil in pipes, intermittent plug/slug flows are multiphase flow regimes often encountered which can create significant pressure fluctuations. To better understand the phenomena and to design equipment for the production and transportation of oil resources, multiphase flow models are essential. Several of these models, for example the slug flow models, apply a number of closure relationships to link gas and liquid phases in a one-dimensional two-fluid model approach. One such closure relationship is the translational velocity for long gas bubbles in liquid flow in pipes. The slug translational velocity is the sum of the bubble velocity in stagnant liquid (i.e. drift velocity) and the maximum velocity in the slug body. Nicklin (1962) proposed the following equation:

$$v_t = C_o v_m + v_d \tag{1}$$

where $C_o$ is approximately 1.2 for turbulent flows and 2.0 for laminar flows, $v_m$ is the mixture velocity (the sum of the superficial liquid and gas velocities), and $v_d$ is the drift velocity.

By considering the potential and kinetic energy only, and ignoring the frictional and capillary effects of the falling liquid around a bubble in a vertical pipe, Dumitrescu (1943) and Davies and Taylor (1950) evaluated the bubble velocity in a liquid in a vertical tube as:

$$v_d = Fr\left[(gD)\left(1 - \rho_g/\rho_l\right)\right]^{1/2} \tag{2}$$

where, $D$ is the pipe diameter, $g$ is the acceleration due to gravity, and $Fr$ is the Froude number, which represents the ratio of inertial to gravitational forces ($Fr = v_d/\left[(gD)\left(1 - \rho_g/\rho_l\right)\right]^{1/2}$). These authors derived the same dimensionless group (Froude number) as 0.351 and 0.328 respectively. By applying the inviscid potential flow theory to steady gravity currents and analysing the problem of an empty cavity advancing along a horizontal pipe filled with liquid, Benjamin (1968) established the Froude number as 0.54. His study, however, ignored the effects of viscosity and surface tension.

Other known set of dimensionless groups that have been applied to

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**Nomenclature and abbreviation**

| | |
|---|---|
| $C_o$ | Constant (in translational velocity) |
| $D$ | Pipe diameter, m |
| $Eo$ | Eötvös Number |
| $Fr$ | Froude Number |
| $Fr_\theta$ | Froude Number at any pipe inclination |
| $g$ | Acceleration due to gravity, m/s$^2$ |
| $Mo$ | Morton number |
| $Re$ | Reynolds number |
| $N_{vis}$ | Viscosity number |
| $v_d$ | Drift velocity, m/s |
| $v_d^h$ | Drift velocity for horizontal flow, m/s |
| $v_d^v$ | Drift velocity for vertical flow, m/s |
| $v_m$ | Mixture velocity, m/s |
| $v_t$ | Translational velocity, m/s |
| Vso | Superficial velocity of oil, m/s |
| $\rho$ | Density, kg/m$^3$ |
| $\mu$ | Viscosity, kgm$^{-1}$s$^{-1}$ |
| $\sigma$ | Surface tension, N/m |
| $\theta$ | Pipe inclination to horizontal, Degree |
| $R$ | Buoyancy Reynolds number |
| LTT | Long Tapered Tail |
| PDF | Probability Density Function |
| STT | Short Tapered Tail |
| STwtB | Short Tapered with detached tiny Bubbles |

estimate the rise velocity of a single bubble moving in liquid in a pipe under the influence of gravitational, inertial, viscous and interfacial forces:

$$\text{Reynolds number,} \quad Re = \frac{\rho v_d D}{\mu} \tag{3}$$

$$\text{Eötvös number,} \quad Eo = \frac{(\rho_l - \rho_g) g D^2}{\sigma} \tag{4}$$

$$\text{Viscosity number,} \quad N_{vis} = \mu \left( g D^3 (\rho_l - \rho_g) \rho_l \right)^{-0.5} \tag{5}$$

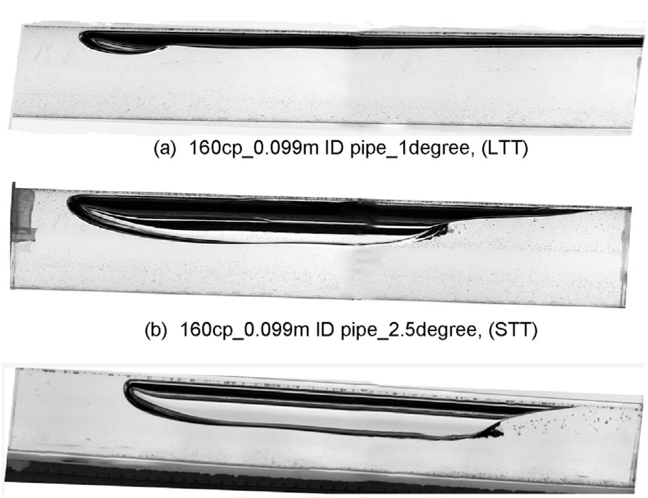
$$\text{Buoyancy Reynolds number,} \quad R = \frac{\left( D^3 g (\rho_l - \rho_g) \rho_l \right)^{0.5}}{\mu} \tag{6}$$

When inertia dominates, $Eo$ is large and $Fr = 0.351, 0.328$ for vertical tubes (Dumitrescu, 1943; Davies and Taylor, 1950 respectively), and for horizontal tubes, $Fr = 0.54$ (Zukoski, 1966; Benjamin, 1968). When the surface tension dominates, the bubbles do not move (Bretherton, 1961; White and Beardmore, 1962; Masica et al., 1964; Weber, 1981). Where viscosity is essential, relationships between *Fr, Eo* and Morton number have been adopted.

Following the pioneer works of Dumitrescu (1943) and Davies and Taylor (1950), many prediction formulae of drift velocity of elongated bubbles in stagnant liquid have been developed (Benjamin, 1968; White and Beardmore, 1962; Brown, 1965; Wallis, 1969; Tung and Parlang, 1976; Weber, 1981; Bendiksen, 1984; Weber et al., 1986; Hasan and Kabir, 1988; Viana et al., 2003; Gokcal et al., 2008; Jeyachandra et al., 2012; and Moreiras et al., 2014). Unfortunately most of the available drift velocity models have applicability limitations and low predictive capabilities, either because they were established using a limited number of experimental data that scarcely account for the combined effects of viscosity, surface tension, and pipe inclination or, because of their formulation. In this study, the drift velocities of elongated bubbles have been gathered from numerous sources and from recent experiments conducted in a low pressure flow loop for a stagnant oil of viscosity 160cP and 1140cP for 0.099 m and 0.057 m internal diameter pipes inclined at angles between 1.0 and 7.5° from horizontal. These data have been used to develop a simplified generalised drift velocity model with high predictive capability. The performance analysis of some of the existing models from the literature is presented first.

## 2. Taylor bubble experimental data

The characteristic shape of an elongated bubble has been suggested by Fagundes et al. (1999) as a means to access the transition between the sub-regimes, slug and plug flow, of intermittent flows. From the recent Taylor bubble experiments conducted in a low pressure flow loop in stagnant and flowing liquid, observed characteristics shapes of the bubbles recorded using high-speed camera are presented on Figs. 1 and 2. In the stagnant liquid (Fig. 1), the Taylor bubble nose always seems to be prolate spheroid (or bell-shaped) and tends to be off the centre, and close to the top of the pipe. However, for the flowing case (Fig. 2), the nose tends to be closer to the centre of the pipe. Depending on the pipe

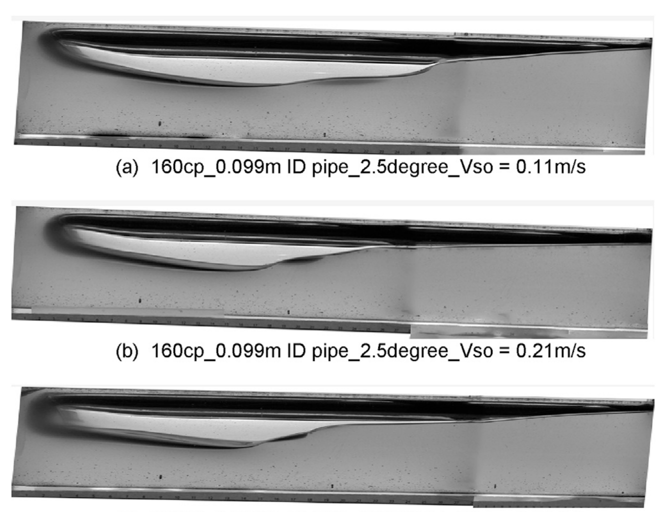


(a) 160cp_0.099m ID pipe_1degree, (LTT)

(b) 160cp_0.099m ID pipe_2.5degree, (STT)

(c) 160cp_0.057m ID pipe_5degree

**Fig. 1.** Some selected bubbles in stagnant liquid conditions.



(a) 160cp_0.099m ID pipe_2.5degree_Vso = 0.11m/s

(b) 160cp_0.099m ID pipe_2.5degree_Vso = 0.21m/s

(c) 160cp_0.099m ID pipe_2.5degree_Vso = 0.31m/s

**Fig. 2.** Some selected bubbles in flowing liquid conditions.