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Numerical study of hydraulic fracture propagation accounting for rock anisotropy

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ABSTRACT

The objective of this study is to investigate the effect of rock anisotropy on hydraulic fracture propagation. The coupled model of rock deformation and fluid flow is established to study hydraulic fracturing of orthotropic formation. Stress field is solved by using the extended finite element method with special tip enrichment functions for orthotropic formation. The coupling between stress field and pressure field is treated by Picard iterative procedure. The modified circumferential tensile stress criterion is used to determine fracture propagation, in which stress intensity factors are determined by an interaction integral method. Numerical results show that when fracture doesn't initiate from the direction of material axis with larger modulus, the fracture propagation direction would change and divert to the direction of material axis with larger modulus. And as Young's modulus ratio between two material axes increases, the phenomenon becomes more obvious. Moreover, shear modulus also enhances the diversion phenomenon of fracture propagation direction. However, the in-situ stress difference could weaken the effect of rock anisotropy. The results indicate that the propagation process of hydraulic fracture is influenced by comprehensive factors including material axis angle, Young' modulus ratio, shear modulus and in-situ stress difference.

1. Introduction

Hydraulic fracturing has become important technology to stimulate production from oil and gas wells. It is also applied to enhance geothermal systems (AbuAisha et al., 2016). On the other hand, rock fracturing is also a crucial issue for geological sequestration of CO_2 and waste disposal. Due to the existence of bedding planes and natural fractures, most rocks often exhibit an inherent material anisotropy. Therefore, it is essential to account for the effect of rock anisotropy on hydraulic fracturing process.

In recent years, lots of hydraulic fracture models have been developed by many researchers (Adachi et al., 2007; Weng, 2015). And different kinds of numerical methods have been used to solve the hydraulic fracturing problem, including finite element method (Advani et al., 1982), extended finite element method, discrete element method (De Pater et al., 2005; Yoon et al., 2015) and displacement discontinuity method (Weng et al., 2011; Zhang et al., 2009; Zeng and Yao, 2016; Yao et al., 2017). The finite element method is widely used in computational mechanics. However when it is used to hydraulic fracturing problem, the mesh needs reconstruction in the simulation. Chen (2012) used cohesive zone element to simulate the fracturing process, but it needs to preset the fracture propagation path. Thus, the extended finite element method is useful to solve the problem, which allows fracture propagates along an arbitrary path without re-meshing (Dolbow and Belytschko, 1999). The application of the extended finite element method has received many interests (Lecampion, 2009; Mohammadnejad and Khoei, 2013; Olson et al., 2009). Gordeliy and Peirce (2013) described coupled algorithms that use the extended finite element method to solve the propagation of hydraulic fractures in an elastic medium. However, these models are restricted to isotropic formations, which may cause great error in anisotropic formations.

Accounting for the effect of anisotropy, the maximum circumferential tensile stress criterion has been modified to consider the difference of critical fracture toughness along two principal material directions (Saouma et al., 1987). This leads to that the propagation process of fracture is very different from that based on assumption of isotropy (Viola et al., 1989). Asadpoure et al. (2006), Asadpoure and Mohammadi (2007)) used the extended finite element method to model crack problem by introducing new crack tip enrichment functions. Wang et al. (2016) simulated hydraulic fracturing in orthotropic formation by using the

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extended finite element method. While in this study, the modified maximum circumferential tensile stress criterion is chosen to determine fracture propagation, and new different results have been obtained.

The paper is organized as follows. Section 2 presents model formulations for hydraulic fracturing problem, including constitutive relations for orthotropic elastic rocks, fracture propagation criterion and fluid flow in the fracture. Section 3 describes details of the numerical algorithms. Stress and pressure fields are solved by using the extended finite element method and standard finite element method respectively. The Picard iterative method is employed to solve the coupling between two fields. In Section 4, one shows the validation of the established model and analyzes effects of material anisotropy angle, elastic modulus ratio, shear modulus and in situ stress difference on fracture propagation. Section 5 draws some concluding remarks.

2. Model formulation

Consider a hydraulic fracture propagating in an anisotropic formation as shown in Fig. 1. The fracture is assumed to initiate in the direction parallel to the maximum principal stress. The injected fluid is Newtonian fluid, and it can be extended to Non-Newtonian fluid easily. The whole mathematic model includes four parts: rock deformation, constitutive relations for orthotropic rocks, fracture propagation criterion and fluid flow, which will be described in detail in the following sections respectively.

2.1. Rock deformation

Rock deformation is caused by the combined effect of remote in-situ stresses and fluid pressure in the fracture. The linear elasticity theory gives the equation of equilibrium

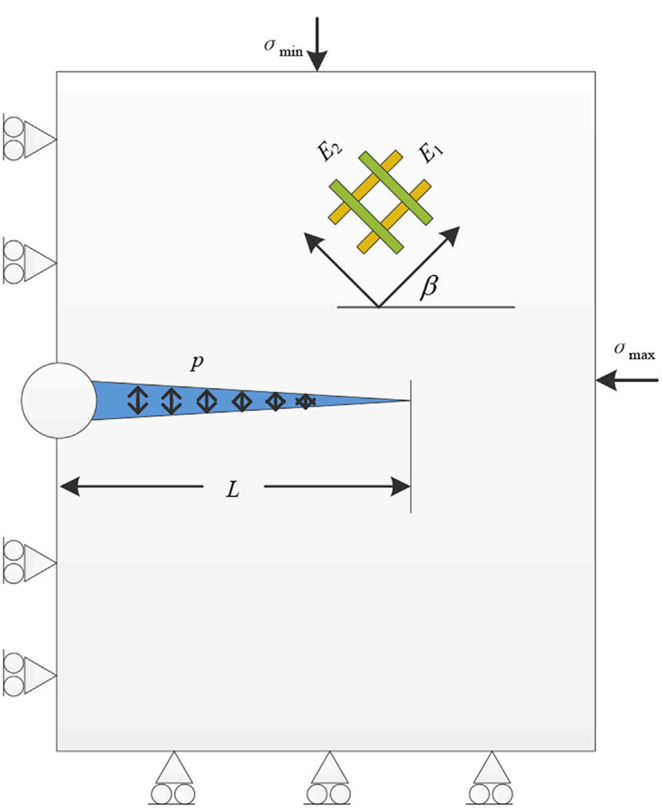

Fig. 1. Physical model of hydraulic fracture propagating in anisotropic formation with material angle β

$$\nabla\cdot\sigma + f = 0 \tag{1}$$

where σ is the Cauchy stress tensor, and f is the body force. The constitutive relations between strain and stress can be simply written with the elastic stiffness matrix or compliance matrix, which will be described in detail for orthotropic formation in the following part.

$$\varepsilon = S : \sigma \tag{2}$$

where ε is the linear strain tensor, and S is the elastic compliance matrix.

Under the assumption of small strains and displacements, the geometric compatibility equation gives

$$\varepsilon(u) = \frac{1}{2}\left(\nabla u + (\nabla u)^T\right) \tag{3}$$

where u is the displacement vector.

The boundaries of studied domain include outer boundary and fracture boundary. Imposed displacements and stresses are prescribed at the outer boundary, and fluid pressure is acting on the fracture surface. The boundary conditions can be given by

$$u|_{\Gamma_g} = g \tag{4}$$

$$\sigma\cdot n|_{\Gamma_h} = h \tag{5}$$

$$\sigma_n|_{\Gamma_f^+} = \sigma_n|_{\Gamma_f^-} = -p(s,t) \tag{6}$$

where Γ_g is the imposed displacement boundary, and g is the imposed displacements. Γ_h is the imposed stresses boundary, and h is the imposed stresses. Γ_f is the fracture surface boundary, and p is the fluid pressure.

2.2. Constitutive elastic relations for orthotropic formation

Horizontal bedding planes are frequently encountered in oil and gas reservoirs, together with vertical natural fractures. This gives rise to that the reservoir layer can be seen as an orthotropic material. With the assumption of linear elastic behaviour, there are nine independent parameters in the elastic stiffness matrix or compliance matrix. The constitutive relations between strains and stresses can be given as

$$\begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{22} \\ \varepsilon_{23} \\ \varepsilon_{31} \\ \varepsilon_{12} \end{bmatrix} = \begin{bmatrix} 1/E_1 & -\nu_{12}/E_2 & -\nu_{13}/E_3 & 0 & 0 & 0 \\ & 1/E_2 & -\nu_{23}/E_3 & 0 & 0 & 0 \\ & & 1/E_3 & 0 & 0 & 0 \\ & & & 1/G_{23} & 0 & 0 \\ & & & & 1/G_{31} & 0 \\ & & & & & 1/G_{12} \end{bmatrix} \times \begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{23} \\ \sigma_{31} \\ \sigma_{12} \end{bmatrix} \tag{7}$$

where ε_{ij} and σ_{ij} are linear strain and stress tensors, respectively. The coefficients in the compliance matrix rely on material parameters in the principal material directions, including Young's modulus, shear modulus and Poisson's ratio.

When the three dimensional geometry is reduced to the two dimensional one, there are two scenarios including plane stress and plane strain. For the plane stress, the constitutive equation in the local coordinate system of principal material direction can be reduced to

$$\begin{bmatrix} \varepsilon_{x'} \\ \varepsilon_{y'} \\ \tau_{x'y'} \end{bmatrix} = \begin{bmatrix} 1/E_1 & -\nu_{12}/E_2 & 0 \\ & 1/E_2 & 0 \\ & & 1/G_{12} \end{bmatrix} \begin{bmatrix} \sigma_{x'} \\ \sigma_{y'} \\ \gamma_{x'y'} \end{bmatrix} \tag{8}$$