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Analytical method for transient flow rate with the effect of the quadratic gradient term

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ABSTRACT

Owing to neglecting the quadratic gradient term (QGT) in the governing equation, the conventional method proposed by van Everdingen and Hurst [Trans. AIME, 1949, 186, 305–324] for obtaining the transient flow rate is inconsistent with material balance and may lead to errors in the prediction of the flow rate for the wells producing at a constant bottomhole pressure. This paper extends the conventional method to consider the effect of the QGT. If the pressure solution in Laplace space under the constant-rate-production condition is known, the flow-rate solution in Laplace space under the constant-pressure-production condition including the effect of the QGT can be determined by the proposed method. Flow-rate solutions for horizontal wells in reservoirs with different lateral outer boundaries under the constant-pressure-production condition are obtained by the proposed method and the conventional method, respectively. The difference between the flow-rate solutions with and without the effect of the QGT is qualitatively and quantitatively analyzed. The proposed method for obtaining the transient flow rate is consistent with material balance, and thus it can be used to obtain more accurate transient flow rate.

1. Introduction

In recent years, well test analysis and rate decline analysis have been widely used to recognize the characteristics of the reservoirs and fluid flows. Two cases, namely the constant-rate-production case and the constant-pressure-production case, have attracted great attention in making the reservoir studies. The bottomhole-pressure solution in the constant-rate-production case and the flow-rate solution in the constant-pressure-production case not only reflect the performance of the wells (Xu et al., 2013; Jia et al., 2015) but also can be employed to do well test analysis (Earlougher, 1977) and rate decline analysis (Blasingame et al., 1991).

During the last decade, many scholars have established a great variety of seepage models and investigated the transient pressure responses for various wells producing at a constant flow rate, such as the vertical wells (Van Everdingen and Hurst, 1949; Ren and Guo, 2014; Moradi et al., 2017), fractured vertical wells (Cinco-Ley and Samaniego, 1981; Berumen et al., 2000; Ren and Guo, 2015a), horizontal wells (Ozkan, 1988; Nie et al., 2012), multiple fractured horizontal wells (Wan and Aziz, 2002; Luo et al., 2014; Ren and Guo, 2015b, 2015c) and so on. However, most of the models do not take into account the effect of the quadratic gradient term (QGT) in the governing equation, which makes the models be inconsistent with material balance and may lead to significant errors in the predicted pressure for the wells producing at a constant flow rate. Recently, much attention has been paid to the transient pressure responses with the effect of the QGT. Some scholars have investigated the pressure behaviors with the effect of the QGT under the constant-rate-production condition for various wells in different reservoir scenarios, such as vertical wells in homogeneous reservoirs (Finjord, 1986), vertical wells in dual-porosity reservoirs (Bai et al., 1994), vertical wells in fractal double-porosity reservoirs (Yao et al., 2012), vertical wells in multiple-zone composite reservoirs (Wang et al., 2013), horizontal wells in homogeneous reservoirs (Lu et al., 2015), multiple fractured horizontal wells with stimulated reservoir volume (Ren and Guo, 2017), and so on.

Compared with the bottomhole-pressure solution in the constant-rate-production case, it is difficult to directly establish models to obtain the flow-rate solution for the well producing at a constant bottomhole pressure, especially when the well type is quite complex. In general, if the bottomhole-pressure solution in Laplace space under the constant-rate-production condition is known, the flow-rate solution in Laplace space under the constant-pressure-production condition can be obtained by the Duhamel's principle (Van Everdingen and Hurst, 1949). Although this method has been widely used to obtain the flow-rate solutions for various

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Nomenclature

Roman symbols

B	volume factor, m^3/m^3
c_f	fluid compressibility, Pa^{-1}
h	formation thickness, m
k	formation permeability, m^2
k_h	horizontal permeability, m^2
k_v	vertical permeability, m^2
L	half-length of horizontal well, m
L_D	dimensionless half-length of horizontal well, dimensionless
p	formation pressure, Pa
p_i	initial formation pressure, Pa
p_w	bottomhole pressure, Pa
q	flow rate under surface conditions, m^3/s
q^*	transformed flow rate, defined in Eq. (15)
q_{Dl}	dimensionless flow rate without the effect of the QGT, dimensionless
q_{Dnl}	dimensionless flow rate with the effect of the QGT, dimensionless
Q	cumulative flow rate under surface conditions, m^3
Q^*	transformed cumulative flow rate, defined in Eq. (25)
r	horizontal radial distance in reservoir formation, $r=\sqrt{x^2+y^2}$, m
r_e	outer boundary radius, m
r_w	wellbore radius, m
s	Laplace transform variable, dimensionless
t	time, s
x, y, z	Cartesian coordinates, m
z_w	vertical coordinate of the well center, m

Greek symbols

α	dimensionless nonlinear flow coefficient, dimensionless
Δp	pressure difference, $\Delta p = p_i - p$, Pa
Δp^*	normalized density change, defined in Eq. (22)
Δq_D	absolute difference between the flow-rate solutions with and without the effect of the QGT, dimensionless
$\Delta\rho$	density difference, $\Delta\rho = \rho_i - \rho$, kg/m^3
δ	relative difference between the flow-rate solutions with and without the effect of the QGT, dimensionless
μ	fluid viscosity, Pa·s
ρ	fluid density, kg/m^3
ρ_i	initial fluid density, kg/m^3
υ	velocity, m/s
ϕ	formation porosity, fraction

Superscript

$-$	Laplace space

Subscript

D	dimensionless

wells in different reservoir scenarios (Ozkan and Raghavan, 1991; Guo et al., 2012; Zhao et al., 2016), the calculated flow-rate solutions fail to consider the effect of the QGT and may result in errors in the prediction of the flow rate for the wells producing at a constant bottomhole pressure. In addition, some scholars (Cao et al., 2004) have directly established the explicit model for vertical wells in the constant-pressure-production case and derived the analytical solutions with the effect of the QGT, but there are also many challenges in obtaining the flow-rate solutions with the effect of the QGT based on the explicit models for complex well types. Inspired by the relationship between the pressure solution and flow-rate solution proposed by van Everdingen and Hurst (1949), it is natural to think that whether there is a similar formula including the effect of the QGT. If the pressure solution in Laplace space under the constant-rate-production condition is known, the flow-rate solution in Laplace space under the constant-pressure-production condition including the effect of the QGT can be determined by the new formula.

In this work, we derive a general flow-rate solution for various wells under the constant-pressure-production condition including the effect of the QGT, and then we take the horizontal well as an example to analyze the characteristics of the flow rate decline with the effect of the QGT. Finally, we make a comparison between the flow-rate solutions with and without the effect of the QGT.

2. Mathematical formulation

In the following, we will focus on deriving the general flow-rate solution under the constant-pressure-production condition for the material-balance flow system which can be described by the nonlinear equation including the QGT. For simplicity and without loss of generality, here we consider a completely penetrating vertical well like the study conducted by van Everdingen and Hurst (1949), but the derived solution is general and suitable for a variety of well types.

We consider the flow of a single-phase liquid (oil or water) in a homogeneous and isotropic medium with the closed top and bottom boundaries. The fluid in the medium has constant compressibility and viscosity.

2.1. Nonlinear governing equation

Considering a two-dimensional radial-flow system, the continuity equation based on the mass conservation is expressed as (Ahmed, 2010)

$$\frac{1}{r}\frac{\partial(r\rho\upsilon)}{\partial r} = \frac{\partial(\rho\phi)}{\partial t}. \tag{1}$$

Fluid flow in the medium is consistent with the Darcy law, which is given as

$$\upsilon = \frac{k}{\mu}\frac{\partial p}{\partial r}. \tag{2}$$

Fluid compressibility is defined by

$$c_f = \frac{1}{\rho}\frac{\partial\rho}{\partial p}. \tag{3}$$

With the assumption of constant permeability, porosity and viscosity, we follow the standard procedure to derive the governing equation as follows (see Appendix A) (Finjord, 1986):

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial p}{\partial r}\right) + c_f\left(\frac{\partial p}{\partial r}\right)^2 = \frac{\phi\mu c_f}{k}\frac{\partial p}{\partial t}. \tag{4}$$

It is observed that Eq. (4) is the nonlinear equation including the QGT, namely the second term on the left side of Eq. (4). If the QGT is negligible, the nonlinear governing equation is simplified as the conventional linear governing equation (Van Everdingen and Hurst, 1949):

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial p}{\partial r}\right) = \frac{\phi\mu c_f}{k}\frac{\partial p}{\partial t}. \tag{5}$$