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# Generation of a Pareto front for a bi-objective water flooding optimization problem using approximate ensemble gradients

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ABSTRACT

Conflicting objectives are frequently encountered in most real-world problems. When dealing with conflicting objectives, decision makers prefer to obtain a range of possible optimal solutions from which to choose. In theory, methods exists that can produce a range of possible solutions, some of which are “Pareto Optimal”. The application of these methods to solve bi-objective production optimization problems is increasing. A recent paper introduced a method to find points on the boundary of the objective function space by solving a constrained optimization problem using adjoint gradients. In this work, we investigate the applicability of using ensemble optimization (EnOpt) (which relies on approximate ensemble gradients instead of exact adjoint-based gradients) to generate points along a “Pareto” front with acceptable computational effort.. Moreover, we investigate the applicability of this approximate gradient technique to solve constrained optimization problems using the augmented Lagrangian method. Finally, we compare the performance of this bi-objective optimization method to a traditional weighted sum method for bi-objective water flooding optimization of two different synthetic reservoir models. The two objectives used in this work are, undiscounted (0%) net present value (NPV), representing long-term targets and highly discounted (25%) NPV, representing short-term operational targets. The controls are inflow control valve (ICV) settings over time for one model and water injection rate controls for the other. The effect of different starting points and the computational efficiency of the constrained optimization method are also investigated.



## 1. Introduction

A majority of studies and applications of life-cycle water flooding optimization using a model-based approach have focused on a single objective optimization with emphasis being placed on the theoretical understanding and practical application of the optimization methodology. Life-cycle optimization essentially aims to find a strategy which optimizes long-term reservoir management targets, but life-cycle optimization is often at the expense of operationally significant short-term targets. Thus, there is a need to solve a bi-objective problem to obtain a strategy that accounts for the two objectives because the long-term perspective is usually in conflict with the short-term targets which are decided by operational constraints, contractual obligations etc. Van Essen et al. (2011) introduced a hierarchical optimization framework to solve such a multi-objective optimization problem. This was motivated by the observation in, e.g., Jansen et al. (2009) that the objective function space consists of many redundant degrees of freedom which can be exploited to optimize a secondary objective. This hierarchical structure provides a single optimal strategy which incorporates multiple objectives. However, decision makers usually prefer to have multiple strategies to choose from, especially when dealing with conflicting objectives. Isebor and Durlofsky (2014) applied an evolutionary algorithm to generate points along a “Pareto” front for a bi-objective water flooding problem. The main pitfall of this approach was the computational effort required to obtain the points on a Pareto front. Also they did not compare the front generated with any other method used to generate Pareto fronts to check if the front obtained was Pareto optimal. Liu and Reynolds (2014) applied the normal boundary intersection method (NBI) first introduced in Das and Dennis (1998) to a bi-objective water flooding problem with and without geological uncertainty. Liu and Reynolds (2014) showed that the NBI method is computationally more efficient than the method of Isebor and Durlofsky (2014) and produces better solutions than the traditional weighted sum method. The NBI method involves

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solving a series of constrained optimization sub-problems. In Liu and Reynolds (2014), these constrained optimization problems were solved using an augmented Lagrangian method using an adjoint formulation to compute the gradients. The adjoint formulation, an overview of which can be found in Jansen (2011) and references therein, is a computationally efficient method which requires access to the simulator source code to implement. Most commercial simulators either do not have a fully developed adjoint code or access to the source code is not permissible. This has led to an increase in the application of various approximate gradient based techniques which are computationally less efficient but use the simulator as a black-box, and are more flexible. Do and Reynolds (2013) provided theoretical connections between various existing approximate gradient techniques which use an ensemble of perturbed controls to estimate a gradient. One such approximate gradient technique introduced in Lorentzen et al. (2006) and thereafter in its current form by Chen et al. (2009) is the ensemble optimization (EnOpt) method. Recently many studies have used EnOpt for life-cycle production optimization problems. Fonseca et al. (2014) applied EnOpt to solve a bi-objective optimization problem using the hierarchical structure proposed by Van Essen et al. (2011). Additionally there has been an increase in the number of applications of different evolutionary algorithms to solve either a bi-objective optimization problem, Isebor and Durlofsky (2014) etc., or for history matching applications, as detailed in Liu and Reynolds (2014). In this work we investigate the applicability of the EnOpt technique to generate points along a "Pareto" front with acceptable computational effort. A secondary aim is the application of EnOpt to solve constrained optimization problems using the augmented Lagrangian method. Note that Fonseca et al. (2014) consider hierarchical optimization (using EnOpt), in which case an a-priory choice is made which of the two objectives is most important. Here we consider bi-objective optimization (using EnOpt) based on the Pareto front approach which provides freedom to the decision maker to choose the relative importance of each of the two objectives, as will be explained in more detail below.

## 2. Theory

This section investigates the applicability of the use of approximate ensemble gradients to calculate points on a Pareto front for bi-objective production optimization problems.

### 2.1. Objective functions

We first define the objective functions followed by an overview of EnOpt. We apply the usual expression for Net Present Value (NPV) as objective function $J$:

$$J = \sum_{k=1}^{K} \left( \frac{\left\{ \left[ (q_{o,k}) \cdot r_o - (q_{wp,k}) \cdot r_{wp} \right] - \left[ (q_{wi,k}) \cdot r_{wi} \right] \right\} \cdot \Delta t_k}{(1+b)^{t_k/\tau_t}} \right), \tag{1}$$

where $q_{o,k}$ is the average oil production rate in $m^3$/day for time step $k$, $q_{wp,k}$ is the water production rate in $m^3$/day for time step $k$, $q_{wi,k}$ is the water injection rate in $m^3$/day for time step $k$, $r_o$ is the sale price of oil in \$/ $m^3$, $r_{wp}$ is the cost of water produced in \$/ $m^3$, $r_{wi}$ is the cost of water injected in \$/$m^3$, $\Delta t_k$ is the length of the $k^{th}$ time step in days, $b$ is the discount factor, $t_k$ is the cumulative time in days corresponding to time step $k$, and $\tau_t$ is the reference time period for discounting, typically one year (i.e. 365 days). In this work the two objective functions are:

– Undiscounted NPV, $b$=0.0 (0%) in Eq. (1), representing the long-term objective ("recovery optimization").
– Highly discounted NPV, $b$=0.25 (25%) in Eq. (1), representing the short-term objective ("day-to-day production optimization").

### 2.2. Ensemble optimization (EnOpt)

In this section, we outline the standard formulation of the EnOpt algorithm as proposed by Chen et al. (2009). We take $\mathbf{u}$ to be a single control vector containing all the control variables to be optimized. This vector has $N$ components where $N$ is equal to the product of the controllable well parameters (number of well settings like bottom hole pressures, rates or valve settings) and the number of control time steps. Chen et al. (2009) sample the initial mean control vector from a Gaussian distribution while, at later iteration steps, the final control vector of the previous iteration is taken as the mean control. However, the initial controls can also be chosen by the user, as will be done in our experiments. The vector of controls is given by,

$$\mathbf{u}_i = [u_1 \quad u_2 \quad \cdots \quad u_N]^T \tag{2}$$

where the counter $i$ preempts the use of multiple control vectors, and where $\mathbf{u}_i$ is assumed to be a random vector which has a mean $\mathbf{u}$ and covariance matrix $\tilde{\mathbf{C}}$, i.e. $\mathbf{u}_i \sim N(\mathbf{u}, \tilde{\mathbf{C}})$. Then an ensemble of $M$ independent samples of N($\mathbf{u}$,$\tilde{\mathbf{C}}$) are generated as,

$$\mathbf{u}_i = \mathbf{u} + \tilde{\mathbf{C}}^{1/2}\mathbf{z}_i, \tag{3}$$

with $i$=1, 2, …, $M$, where $\mathbf{z}_i \sim N(\mathbf{0},\mathbf{I})$, i.e., each $\mathbf{z}_i$ is a vector of independent standard random normal deviates, and $\tilde{\mathbf{C}}^{1/2}$is any square root of $\tilde{\mathbf{C}}$. In our examples $\tilde{\mathbf{C}}^{1/2} = \mathbf{L}$, where $\mathbf{L}$ is the lower triangular matrix in the Cholesky decomposition of $\tilde{\mathbf{C}}$. We truncate any element of the ensemble of controls outside of the set of bounds to the bound value. Then, the sample mean is computed as

$$\bar{\mathbf{u}} = \frac{1}{M} \sum_{i=1}^{M} \mathbf{u}_i. \tag{4}$$

To estimate the gradient, a mean-shifted ensemble matrix is defined as

$$\Delta \mathbf{U} = [\mathbf{u}_1 - \bar{\mathbf{u}} \quad \mathbf{u}_2 - \bar{\mathbf{u}} \quad \cdots \quad \mathbf{u}_M - \bar{\mathbf{u}}]. \tag{5}$$

Similarly, a mean-shifted objective function vector is defined as

$$\Delta \mathbf{j} = \left[ J(\mathbf{u}_1) - \bar{J} \quad J(\mathbf{u}_2) - \bar{J} \quad \cdots \quad J(\mathbf{u}_M) - \bar{J} \right]^T, \tag{6}$$

where the average of the objective function is given by

$$\bar{J} = \frac{1}{M} \sum_{i=1}^{M} J(\mathbf{u}_i). \tag{7}$$

In the present paper, we use as the search direction in a steepest ascent algorithm an approximation to the gradient, rather than the approximation of the smoothed gradient that is used in standard EnOpt. The approximate gradient is

$$\mathbf{g} = (\Delta \mathbf{U} \Delta \mathbf{U}^T)^{\dagger} \Delta \mathbf{U} \Delta \mathbf{j} = (\Delta \mathbf{U}^T)^{\dagger} \Delta \mathbf{j}, \tag{8}$$

where the superscript † indicates the Moore-Penrose pseudo inverse, which is conveniently computed using a singular value decomposition (SVD); see, e.g., Strang (2006). Do and Reynolds (2013) demonstrated that it is akin to what is known as a 'simplex gradient', Conn et al. (2009). They also provided theoretical connections between various ensemble methods such as simultaneous perturbation stochastic approximation (SPSA), simplex gradient and EnOpt. Moreover, they proposed a modification to the gradient formulation which uses the current control vector $\mathbf{u}^{\ell}$and the corresponding objective function value $J^{\ell}$ to calculate