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Localized model order reduction in porous media flow simulation

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ABSTRACT

This paper introduces a new approach to construct an efficient reduced order model for fluid flow simulation and optimization in porous media. For nonlinear systems, one of the most common methodology used is the proper orthogonal decomposition (POD) combined with discrete empirical interpolation method (DEIM) due to its computational efficiency and good approximation. Whereas regular POD-DEIM approximates the fine scale model with just one single reduced subspace, the localized POD (LPOD) and localized DEIM (LDEIM) are constructed by computing several local subspaces. Each subspace characterize a period of solutions in time and all together they not only can approximate the high fidelity model better, but also can reduce the computational cost of simulation. LPOD and LDEIM use classification approach to find these regions in the offline computational phase. After obtaining each class, POD and DEIM is applied to construct the basis vectors of the reduced subspace. In the online phase, at each time step, the reduced states and nonlinear functions will be used to find the most representative basis vectors for POD and DEIM without requiring fine scale information. The advantages of LPOD and LDEIM are shown in a numerical example of two phase flow in porous media.

1. Introduction

Reservoir management enables one to obtain the most favorable production scenario given the current information of the reservoir, such as reservoir characterization parameters (permeability, porosity) and production data. Very often, one is directed to computational methods (reservoir simulation) to develop new techniques or to optimize the existing fields. There are two main challenges in this realm, the large amount of computational infrastructure necessary to perform such calculations; and the fact that there is no certainty in the range of parameters. In this paper our primary focus is to tackle the first problem by means of reduced order models. Note that the latter issue can be investigated by uncertainty quantification and parameter estimation.

In many cases, reduced-order modeling techniques have shown to be a viable way of mitigating computational complexity in simulation of large-scale reservoirs, while maintaining high level of accuracy. The options range from non-intrusive methods that do not depend on modifications of a reservoir simulation code (Cardoso and Durlofsky, 2010; Lerlertpakdee et al., 2014; Ghommem et al., 2015), to more intrusive and sophisticated methods that depend on several modifications of legacy code or the development of new simulator codes (Heijn et al., 2004; Gildin et al., 2013; Gildin and Ghasemi, 2014).

POD is one of the efficient intrusive methodology used in model order reduction context due to its computational simplicity and good approximation (Van Doren et al., 2004). However, computational savings are not always attained because in order to evaluate nonlinear terms, the reduced state needs to be projected back to fine scale state yielding similar computational cost as the original system. There are different techniques to alleviate this problem. One approach is to linearize these nonlinear functions (trajectories) around several known states and use these piecewise linear solutions, see Cardoso and Durlofsky (2010). Ghasemi and Gildin (2015) used quadratic bilinear formulation to reformulate nonlinear saturation function into bilinear form and then applied model order reduction. Here, we use discrete empirical interpolation method (DEIM), where one constructs another subspace for reducing the nonlinear function evaluations (Chaturantabut and Sorensen, 2010).

In reservoir simulation and optimization, POD-DEIM can yield several orders of magnitude reduction in the size of models and reduce the computational cost significantly (Ghasemi et al., 2015). However, if the system exhibits a very dynamic state with a wide range of changes, many POD and DEIM basis vectors are required to accurately approximate the states of a system as well as the nonlinear terms. A remedy to this problem is to search for not just a single global reduced subspace, but rather multiple local subspaces in time in order to update the bases online, i.e during the reconstruction of the results under different boundary conditions by running a reduced order model. Thus, in this work we refer to

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this procedure as localized model order reduction.

In this technique rather than just one single subspace, multiple local subspaces are constructed for the reduced system. In the online phase, based on the state of the system at each time step, the proper subspace that has the best approximation will be selected. By applying the localization technique, the dimension of the reduced subspace at each time step is decreased more (compared to having a single set of global bases for the entire simulation time) and consequently the computational cost is reduced even further.

The localization idea was introduced for POD in Amsallem et al. (2012) and was verified in reducing the pressure state in dynamical equations describing a MEMS switch device. They suggested using auxiliary quantities to ensure an online selection procedure is independent of the original system dimension. However, the number of auxiliary quantities, which are computed in the offline phase, scales cubically with the number of clusters. This can be an issue especially in a optimization workflow, whereby one updates bases of reduced model periodically to obtain more stable and accurate solution. Following localized POD (LPOD), localized DEIM (LDEIM) was proposed in Peherstorfer et al. (2014) based on machine learning techniques and using efficient classification algorithm. They discussed both parameter and state based LDEIM approaches and applied it to a steady reacting flow simulation.

In this paper, we extend LPOD and LDEIM to reduce the computational cost of porous media fluid flow simulation. We present a new approach to resolve the existing issues in LPOD by reducing the number of required auxiliary parameters. Also, an efficient reduced order model for the fluid transport equation is presented by applying LDEIM on the nonlinear function of fractional flow.

2. Two-phase flow model

In this section, we summarize the underlying partial differential equations related to porous media flow simulation. In particular, we briefly discuss two-phase oil-water systems, see Aarnes et al. (2009) for more details.

We consider two-phase flow in a reservoir domain under the assumption that the displacement is dominated by viscous effects; i.e., we neglect the effects of gravity, compressibility, and capillary pressure. The two phases are water and oil, and they are assumed to be immiscible. The Darcy's law for each phase is as follows,

$$v_l = -\frac{k_{rl}(s)}{\mu_l} K \nabla p, \tag{1}$$

where v_l is the phase velocity, K is the permeability tensor, k_{rl} is the relative permeability to phase l ($l = o, w$), s is the water saturation (we use s instead of s_w for simplicity) and p is pressure. Throughout the paper, we will assume that a single set of relative permeability curves is used. Combining Darcy's law with conservation of mass allows us to express the governing equations in terms of the so-called pressure and saturation equations as,

$$-\nabla\cdot\big(\lambda(s)K\nabla p\big) = q_w + q_o, \tag{2}$$

$$\phi\frac{\partial s}{\partial t} + \nabla\cdot\Big(f_w\Big(s\Big)v\Big) = \frac{q_w}{\rho_w}, \tag{3}$$

where ϕ is the porosity, λ is the total mobility defined as,

$$\lambda(s) = \lambda_w(s) + \lambda_o(s) = \frac{k_{rw}(s)}{\mu_w} + \frac{k_{ro}(s)}{\mu_o}, \tag{4}$$

$f_w(s)$ is the fractional flow function,

$$f_w(s) = \frac{\lambda_w(s)}{\lambda(s)} = \frac{k_{rw}(s)}{k_{rw}(s) + \frac{\mu_w}{\mu_o}k_{ro}(s)}, \tag{5}$$

and v is the total velocity defined as,

$$v = v_w + v_o \tag{6}$$

In this paper, we follow a sequential formulation; at each time step one solves for pressure and flux first and then use these results to solve for saturation. We employ mixed finite element methods (Brezzi and Fortin, 2012) to discretize the pressure equation in order to use the velocity field and preserve the mass conservation, see Ghasemi et al. (2015) for more details and examples. One can write the pressure equation after spatial discretization of the problem as the following system of equations

$$\begin{pmatrix} B(\lambda(s)) & -C^T \\ C & 0 \end{pmatrix}\begin{pmatrix} v \\ p \end{pmatrix} = \begin{pmatrix} 0 \\ g \end{pmatrix} \tag{7}$$

where matrix B and C are derived from FEM discretization and g results from the sink/source terms, see Appendix A for derivation.

Generally, an implicit time (backward Euler) discretization will be applied to solve for saturation profile, while a mass conservative finite volume is used for the spatial derivative discretization in saturation equation. Consider a cell Ω_i with interfaces γ_{ij} and associated normal vectors n_{ij} pointing out of Ω_i, the saturation Eq. (3) will be discretized as,

$$\frac{\phi_i}{\Delta t}\Big(s_i^k - s_i^{k-1}\Big) + \frac{1}{|\Omega_i|}\sum_{j\neq i} F_{ij}\Big(s_i^k\Big) = \frac{q_w\big(s_i^k\big)}{\rho} \tag{8}$$

where s_i^k is the cell-average of the water saturation at time t^k, and F_{ij} is the numerical approximation of the flux over interface γ_{ij},

$$F_{ij} \approx \int_{\gamma_{ij}} f_w(s)_{ij}\Big[v_{ij}(s)\Big].\, n_{ij}dv \tag{9}$$

Note that to find $f_w(s)_{ij}$ over each interface, there are different schemes. It is common to use first order approximation, known as upwinding method, defined as follows (Aarnes et al., 2009),

$$f_w(s)_{ij} = \begin{cases} f_w\big(s_i\big) & \text{if } v.\, n_{ij} \geq 0 \\ f_w\big(s_j\big) & \text{if } v.\, n_{ij} < 0 \end{cases} \tag{10}$$

Thus, neglecting capillary pressure and gravity, one can derive the following differential equation to solve for saturation,

$$s^k = s^{k-1} + A(v^k)f\big(s^k\big) + q^+ \tag{11}$$

where

$$q_i^+ = \frac{1}{\phi_i\Omega_i}\max\Big(q_i, 0\Big) \tag{12}$$

Note that the pressure and velocity equation in Eq. (7) is a linear system, whereas the saturation equation in Eq. (11) is nonlinear system and can be solved for s^k by iterative methods such as Newton-Raphson efficiently. To do so, the residual as a function of saturation is defined as follows,

$$R(s) = s - s^{k-1} - A(v^k)f\Big(s\Big) + q^+, \tag{13}$$

and the Jacobian as,

$$J(s) = \frac{\partial R}{\partial s} = \mathrm{I} - A\, f'(s). \tag{14}$$

where $f'(s)$ is the first derivative of the fractional flow with respect