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# Analyzing variable-rate flow in volumetric oil reservoirs

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ABSTRACT

Estimating average-reservoir pressure ($p_{av}$) and its evolution with time is critical to analyzing and optimizing reservoir performance. Normally, selected wells are shut in periodically for buildup tests to determine $p_{av}$ over time. Unfortunately, shutting-in wells leads to loss of production. Today, however, real-time surveillance—the continuous measurement of flowing pressures and rate data from the oil and gas wells—offers an attractive alternative technique to obtain average-reservoir pressure while avoiding loss of revenue.

A direct method for estimating $p_{av}$ from flowing pressures and rate data is available. However, the method is for an idealized case that assumes constant production rate during pseudosteady-state (PSS) flow, which is generally untrue for real wells. This paper extends that approach so that it can be used to analyze field data with variable rates/variable pressures during boundary-dominated flow (BDF). This approach is based on a combination of rate-normalized pressure and superposition-time function. The mathematical basis is presented in support of this approach, and the method is validated with synthetic examples and verified with field data.

This modified approach is used to estimate average-reservoir pressure that uses flowing pressures and production rates during BDF, allowing the classical material balance calculations to be performed. These calculations, in turn help determine the reserves, recovery factor, and reservoir drive mechanisms, allowing the reservoir performance and management to be properly evaluated. Furthermore, this method can be used to calculate both connected oil volume and reservoir drainage area as a function of time. Finally, this approach provides a reasonable estimation of the reservoir's shape factor.



## Introduction

Average reservoir pressure is one of the essential parameters in reservoir-engineering calculations. Knowledge of evolving $p_{av}$ is required when estimating in-place hydrocarbon volumes, leading to estimation of its recovery. Moreover, the continuous monitoring of $p_{av}$ with time is needed to ascertain reservoir behavior and optimize the reservoir-performance evaluation.

Traditionally, wells are shut-in for buildup testing to estimate the average-reservoir pressure, but this practice results in loss of production. To avoid the lengthy shut-in tests; various techniques have emerged in the literature to estimate $p_{av}$ from both flowing pressure and rate data. Mattar and McNeil (1998) presented the concept of flowing material-balance method for the constant-rate case. Mattar et al. (2006) then extended this technique to handle the variable rate situation using the concept of material-balance time ($t_{mb}$), introduced earlier by Blasingame and Lee (1986). Recently, Ismadi et al. (2012) showed the use of combined static- and dynamic-material-balance methods to arrive at the same solution for in- place volume in gas reservoirs. Medeiros et al. (2010) proposed the transient–PI method to estimate $p_{av}$ as a function of time. In yet another method, Kabir et al. (2012) demonstrated that the transient flow-after-flow testing could be also used to estimate the average-reservoir pressure, regardless of well location within a drainage boundary and reservoir layering.

Agarwal (2010) combined the PSS flow equation with the material-balance equation to relate $p_{av}$ and the bottomhole flowing pressure, $p_{wf}$. Agarwal (2010) used prime- and log-derivatives to distinguish between the flow regimes. The derivation of Agarwal (2010) method is based on the assumption of constant rate during the PSS flow period. Unfortunately, the constant-rate case during the PSS period is an idealized situation. Real field data is naturally in variable-rate/variable-pressure mode during the BDF. Note that BDF implies that pressure perturbations due to production have reached all reservoir boundaries in variable-rate situations, whereas the PSS flow is tied to constant-rate production.

This study extends Agarwal (2010) approach, so that it can be

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**Nomenclature**

| | |
|---|---|
| $A$ | drainage area, $ft^2$ ($L^2$) |
| $B$ | oil formation volume factor, RB/STB ($L^3$/ $L^3$) |
| $c_t$ | total compressibility, 1/psi ($Lt^2$/m) |
| $C_A$ | reservoir shape factor, dimensionless |
| $h$ | reservoir net pay thickness, ft (L) |
| $k$ | effective permeability to oil, md |
| $N$ | initial oil in place, STB ($L^3$) |
| $N_p$ | cumulative oil produced, STB ($L^3$) |
| $p_i$ | initial reservoir pressure, psi (m/$Lt^2$) |
| $p_{av}$ | average-reservoir pressure, psi (m/$Lt^2$) |
| $p_{wf}$ | bottomhole flowing pressure, psi (m/$Lt^2$) |
| $\Delta p_{wf}$ | ($p_i$–$p_{wf}$), psi (m/$Lt^2$) |
| $p_D$ | dimensionless pressure, $p_D = \frac{kh\Delta p}{141.2qB\mu}$ |
| $q$ | oil flow rate, STB/D ($L^3$/t) |
| $q_n$ | oil flow rate at the $n^{th}$ time period, STB/D ($L^3$/t) |
| $\Delta q_j$ | ($q_j - q_{j-1}$), STB/D ($L^3$/t) |
| $r_w$ | wellbore radius, ft (L) |
| $S_o$ | oil saturation, fraction |
| $s$ | skin factor, dimensionless |
| $t$ | producing time, hours (t) |
| $t_{mb}$ | $N_p$/q(t), days (t) |
| $t_{DA}$ | dimensionless time based on drainage area, $A$, $t_{DA} = 2.637 \times 10^{-4}\frac{kt}{\varphi(\mu c_t)_i A}$ |
| $ß$ | difference between the rate-normalized pressure and normalized-log derivative during BDF |
| $\Delta$ | difference |
| $\mu$ | oil viscosity, cp |
| $\phi$ | porosity, fraction |

applied to the variable-rate case. First, we summarize the Agarwal (2010) method for the constant-rate case. Next, we present the modified Agarwal approach by coupling the rate-normalized pressure with the superposition-time function. Finally, the modified approach is validated with synthetic examples and verified with field data.

## Summary of Agarwal's constant-rate method

Agarwal (2010) method uses flowing pressure and rate data collected from oil and gas wells during the PSS flow period to estimate average-reservoir pressures. We summarize his method here to establish a starting point. Agarwal (2010) incorporated the transient and PSS flow equations with the material-balance equation to relate $p_{av}$ and $p_{wf}$, the flowing bottomhole pressure. He used the concept of prime derivative and log derivative (Bourdet derivative) under the main assumption of constant-production rate during PSS flow conditions.

### *Prime and log derivatives*

Prime derivatives are those variables ($p_D$ and $\Delta p$) that are differentiated directly with respect to either dimensionless time ($t_{DA}$) or real time ($t$). Log derivatives are those variables ($p_D$ and $\Delta p$) that are differentiated with respect to a natural log of either dimensionless time or real time. Each derivative provides useful insight into the behavior of both transient and PSS flow regimes. Table 1 summarizes the prime and log derivative during transient and PSS flow conditions.

### *Characteristics of prime and log derivatives*

Fig. 1 shows the prime and log derivatives as functions of time. The characteristics of both plots are the following

1. The log–log plot of prime-derivative versus $t_{DA}$ yields a straight line with negative slope during the transient flow period; thereafter, its value becomes constant and equals to $2\pi$ during the BDF period after a short transition period.

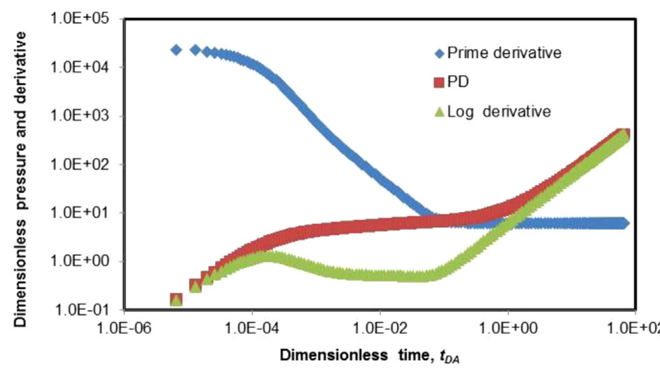


**Fig. 1.** Variation of $p_D$, prime and log derivatives with dimensionless producing time for a well in center of a square-drainage boundary.

2. The log–log plot of log-derivative versus $t_{DA}$ results in a constant value of 0.5 during the transient period. Thereafter, a positive unit-slope line develops to signify the PSS flow, with a transition period in between the two flow regimes. Fig. 1 presents both types of derivatives, which complement each other. Note that the transition period associated with the prime and log derivatives is highly dependent on the reservoir configuration and position of the well with respect to reservoir boundaries.

3. As shown in Appendix A, a comparison of the right side of the log-derivative during PSS flow (Eq. (A-7)) and the material-balance form Eq. (A-3) reveals that both have the same expression during the PSS flow period. In other words,

$$p_D{}'(t_{DA}) = \frac{dp_D}{d\ln t_{DA}} = 2\pi t_{DA} \tag{1}$$

$$p_{Dmb}(t_{DA}) = \frac{kh(p_i - p_{av})}{141.2qB\mu} = 2\pi t_{DA} \tag{2}$$

These equations suggest that the material-balance equation is identical to the log-derivative during PSS flow period as given by the following expression:

**Table 1**
Prime and log derivatives.

| Type of Flow Regime | Prime derivative | Log derivative |
|---|---|---|
| Transient | $p'_{D_p}(t_{DA}) = \frac{dp_D}{dt_{DA}} = 0.5(\frac{1}{t_{DA}}) = 0.5(t_{DA})^{-1}(1)$ | $p_D{}'(t_{DA}) = \frac{dp_D}{d\ln t_{DA}} = 0.5(2)$ |
| Pseudosteady-state (PSS) | $p'_{D_p}(t_{DA}) = \frac{dp_D}{dt_{DA}} = 2\pi(3)$ | $p_D{}'(t_{DA}) = \frac{dp_D}{d\ln t_{DA}} = 2\pi t_{DA}(4)$ |