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A rapid and accurate method for calculation of capillary pressure from centrifuge data

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ABSTRACT

Centrifugation technique is among various methods for rock capillary pressure measurement. In this method, capillary pressure data is not directly measured and it should be calculated from production data. Calculation of capillary pressure data needs conversion of the measured fluid productions to the local saturation values. Considering frequent use of the conventional centrifuge method in the industry, an accurate and rapid technique is highly demanded. Several interpretation techniques have been reported in the literature. Simple methods can be found with less accuracy, while accurate methods usually require time consuming analysis. In this paper, a simple method is proposed based on a semi-analytical solution for calculation of local saturation from experimental data. In this method fundamental centrifuge equation, which is a Voltera equation, is converted by Laplace transformation. Inverse Laplace transformation is performed assuming a sub-polynomial equation for each segment of average saturation ($\bar{S}$). The performance of the proposed method is evaluated using two theoretical functions for capillary pressure. The results obtained by new method are compared with the results from the available methods in the literature.

1. Introduction

Capillary pressure is defined as pressure difference across the interface between two immiscible fluids. It is known as a crucial factor controlling the fluid distributions in reservoir rock. Capillary pressure can be measured as a function of the wetting phase saturation in laboratory by using porous plate method, mercury injection, centrifugation techniques and etc. The centrifuge technique, for capillary pressure measurement, was introduced by Hassler and Brunner (1945) and Slobod et al. (1951). In a drainage centrifuge experiment, a liquid-saturated core, confined in a special holder, is spun at different rotational speeds. The core holder contains another fluid which can replace the liquid inside the core. After equilibrium at each speed, the amount of liquid expelled from the core is measured. Then the average saturation ($\bar{S}$) inside the core can be easily calculated using the pore volume and the produced liquid. It should be noted that as the centrifugal force varies along the core (Fig. 1), the capillary pressure and wetting phase saturation change along the core.

As shown in Fig. 1, the capillary pressure is at maximum level at the inlet (Pc_1) and it is decreased to zero at the outlet where the wetting saturation is at maximum value. Since the average saturation ($\bar{S}$) can be only measured from the experimental data, the local saturation (S) should be calculated based on the conversion methods. Hassler and Brunner (1945) and Hermansen et al. (1991) have shown that the average saturation $\bar{S}(Pc_1)$ and local saturation $S(Pc)$ can be related to each other. In a drainage experiment, the capillary pressure at any distance from the center can be stated as

$$Pc_r = \frac{1}{2}\Delta\rho\,\omega^2\,(r_2^2 - r^2) \quad (1)$$

where r is the distance from the centrifuge axis; ω is the rotation speed; $\Delta\rho$ is the difference between the phase densities. Based on Eq. (1), capillary pressure at the inlet face is

$$Pc_1 = \frac{1}{2}\Delta\rho\,\omega^2\left(r_2^2 - r_1^2\right). \quad (2,a)$$

If the density difference is given in gr/cm^3, ω in rpm, r_1 and r_2 in cm, then the inlet face capillary pressure in psi can be calculated as follows:

$$Pc_1 = 7.953\times10^{-8}\,\Delta\rho\,\omega^2\left(r_2^2 - r_1^2\right). \quad (2,b)$$

Using Eqs. (1) and (2),

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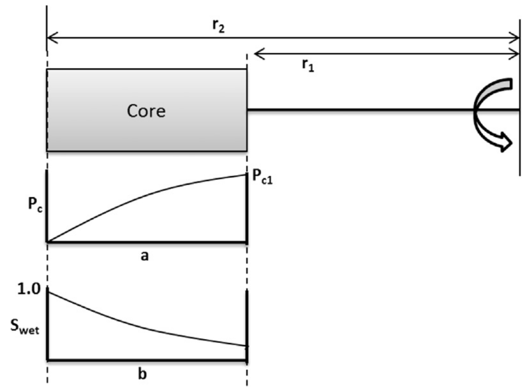

Fig. 1. Local saturation and local capillary pressure along the core during the drainage centrifuge experiment.

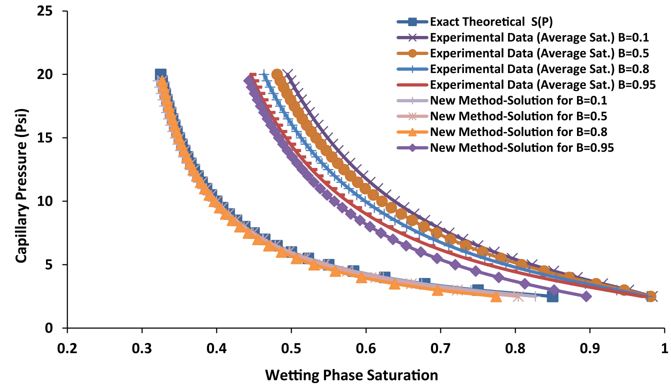

Fig. 2. Calculated saturations from an artificially generated centrifuge data; the theoretical capillary pressure was used to generate experimental data for different B values.

$$Pc = Pc_1\frac{(r_2^2 - r^2)}{(r_2^2 - r_1^2)}. \tag{3}$$

It is evident that the capillary pressure is not constant along the core. Average saturation can be defined as follows:

$$\bar{S} = \frac{1}{(r_2 - r_1)}\int_{r_1}^{r_2} S(r)dr. \tag{4}$$

By substituting and rearranging, the following equation can be derived:

$$\bar{S}(Pc_1) = \frac{r_1 + r_2}{2\sqrt{Pc_1}}\int_0^{Pc_1}\frac{S(Pc)}{\sqrt{r_2^2\,Pc_1 - \left(r_2^2 - r_1^2\right)Pc}}\,dPc, \tag{5}$$

or

$$\bar{S}(Pc_1) = \frac{r_1 + r_2}{2r_2\sqrt{Pc_1}}\int_0^{Pc_1}\frac{S(Pc)}{\sqrt{Pc_1 - B\,Pc}}\,dPc, \tag{6}$$

where

$$B = \frac{\left(r_2^2 - r_1^2\right)}{r_2^2} = 1 - \left(\frac{r_1}{r_2}\right)^2. \tag{7}$$

Eq. (6) is known as fundamental centrifuge equation. For drainage and imbibition experiments, centrifuge data ($\bar{S}(Pc_1)$) can be converted to capillary pressure data ($S(Pc)$) using Eq. (6). There are basic assumptions in developing this equation. The main assumptions are the hydrostatic equilibrium at each phase and the boundary condition of $Pc = 0$ at the out flow. In addition, it is assumed that one-dimensional centrifugal force field exists inside the core plug (assumption of linear field distribution; Christiansen, 1992; Forbes, 1997a, 1997b). Furthermore, there are other physical assumptions such as no cavitation, end-piece effects, homogeneity of the core, etc. All these assumptions were reviewed by Hirasaki et al. (1988) and O'Meara et al. (1992).

Many researchers have proposed numerous approximate solutions for Eq. (6). The Hassler and Brunner (HB) solution (Hassler and Brunner, 1945) is one of the earliest solution which is being used by industry frequently. They assumed that the pressure field in the core is linear and the gravity effect is negligible. These assumptions can be accepted only for short and narrow samples rotated far from the rotation axis. As demonstrated by Forbes (1997a), this method gives smaller saturations compared to the true solution ($S(Pc)$). This usually happens as the parameter B is usually far from zero. There are other solutions which address the fundamental equation (Eq. (6)) while neglecting gravity and radial effect (M=0 and N=0 respectively). The Hoffman solution (Hoffman, 1963), van Domselaar solution (van Domselaar, 1984), Rajan solution (Rajan, 1986) and Forbes first and second methods (Forbes, 1994) are the most popular and simple methods. In addition, other techniques also have introduced such as Clinch approximate solution (Clinch, 1998), Chen and Ruth solution (Chen and Ruth, 1993), Ruth and Wong first and second solution (Ruth and Wong, 1988, 1990) and Hermansen solution (Hermansen et al., 1991). Implementation of some proposed solutions requires different algorithms and numerical schemes. The results obtained from each solution can be significantly different depends on the applied implementation scheme (see Forbes, 1997a). However, it is noted that all mentioned solutions address a simplified forms of Eq. (6) and not the complete equation. It is because the Eq. (6) (which is a Voltera equation) is known to be ill-conditioned (Hermansen et al., 1991). In other words, the uncertainties on saturation measurements are known to be increased when inverting this equation. In a survey performed by Society of Core Analysis (Forbes, 1997a) different interpretation procedures were studied and the assumptions, source of errors and performances were investigated. Subbey and Nordtvedt (2002) used a semi-iterative regularization method for solving the integral equation which needs much mathematical computation. Ferno et al. (2009) proposed the Nuclear Tracer Imaging Centrifuge (NTIC) method which measures the fluid saturation profile across the core to obtain the capillary pressure curve directly. However this method can be accurate, but it needs special experimental facilities. Among all proposed methods, it is believed that the Forbes methods are the most accurate and simple technique to use for interpretation of conventional centrifuge data to obtain capillary pressure. However Forbes methods (1994) are simple to implement, but the method generates fluctuation when using different size pressure steps (discussed in section 4). Considering frequent use of the conventional centrifuge method in the industry, an accurate and rapid technique is still demanded. In this paper, a simple and accurate method is proposed for calculating the local saturation from experimental data. In this technique fundamental centrifuge equation, which is a Voltera equation, is converted by Laplace transformation. Inverse Laplace transformation was performed assuming a sub-polynomial equation for each segment of the average saturation function ($\bar{S}$). Then the correction scheme is also proposed to improve the results when the simplified assumption is not valid.

2. Laplace transformation

Volterra integral equations is an integral equation of the form

$$\int_0^t \zeta(t, r)\phi(r)dr = \psi(t), \quad t \geq 0, \tag{8}$$