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Critical buckling of drill strings in curvilinear channels of directed bore-holes

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ABSTRACT

The problem about theoretic modeling incipient stage of a drill string (DS) buckling in a curvilinear channel of a deep directed bore-hole is stated. On the basis of the theory of curvilinear flexible rods, nonlinear differential equations describing elastic bending of the DS inside the bore-hole cavity are deduced. The effects of the DS curvature, its axial non-uniform prestressing, action of torque, and interaction between the DS and the bore-hole surface are taken into consideration. Owing to the use of curvilinear coordinates in the constraining channel surface and a specially chosen concomitant reference frame, it became possible to separate the desired variables and to reduce the total order of the equations system. With the aim to analyze critical states of the DS equilibrium and to construct modes of its stability loss, these equations are linearized in the vicinity of the considered state and eigen-value problem is formulated. Techniques for its numerical solution are proposed. Via its use the critical loads and shapes of buckling DSs are found for their different locations inside the bore-hole with circular trajectory. It is shown that the buckling proceeds with generation of irregular harmonic wavelets localized inside the well or in the neighborhood of its boundaries. Influence of boundary conditions, torque, clearance value, and channel axis curvature on the buckling process is analyzed. The presented results illustrate the applicability of the method proposed for the regimes of drilling and the drill string lowering and raising.

1. Introduction

At the present time, in politics, business, and oil-gas industry, the sharp-plotted scenarios connected with extraction and redistribution of hydrocarbon fuels are played. Additional freshness is contributed into this atmosphere by "shale revolution" broken out owing to development of new technologies of curvilinear bore-hole drilling. Inasmuch as the inclined and horizontal bore-holes penetrate the oil- and gas-bearing strata along the laminated structure of the underground deposits, they cover larger zones of fuel output and are effective expedients to enlarge the extraction efficiency (Brett et al., 1989; Aadnoy and Andersen, 2001; Choe et al., 2005; Sawaryn et al., 2006). By way of example, as early as 10 years ago, only one-third of the hydrocarbon fuels contained in the oil layer could be extracted. Now, the modern technologies make it possible to enlarge this index to 70%. In addition to that, the deposits hitherto considered economically unprofitable became successful owing to the new means of drilling.

The most ill effects associated with buckling the DS inside curvilinear bore-holes consist in deterioration of conditions of DS – bore-hole interaction leading to essential enlargement of friction forces, resulting the impossibility to transfer the required axial force to the bit and the DS lock-up situation. Therefore, the problems on theoretical simulation of the phenomena of critical and post-critical buckling of DSs in vertical and curvilinear bore-holes are the issues of current importance. In spite of the semicentennial history, beginning from their first analysis in vertical wells by Lubinski et al. (1962) till now and paper by Dawson and Paslay (1984), dedicated to DS buckling in inclined bore-holes, they are far from completion. Detailed analysis of their state is presented by Cunha (2004), Mitchell (2008) and Mitchell and Samuel (2009). It follows from these reviews that, mainly, the approaches used by researchers attacking this problem were based on the buckling mode approximation by sinusoidal or helical curves (as Euler did in 1744). Nevertheless, Cunha (2004): "One important point noted in this literature review is that different authors have presented what could be seen as conflicting results for critical forces of buckling".

Mitchell (2008) emphasizes: "The original buckling analysis by Lubinski et al. (1962) has inspired 4 (now 5; *authors*) decades of work to understand the many aspects of tubing buckling. More than 40 (now 50; *authors*) years later, we find that there are still challenging problems to solve and difficult questions to answer". Some of them include

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(1) What is the critical buckling load in curved, 3D bore-holes?
(2) How does torque affect the critical buckling load?
(3) What is the buckling mode in them?
(4) What effect does friction play in DS buckling?
(5) How do the boundary conditions affect the buckling process?

Not in vain, Mitchell (2008) titled his paper: "Tubing Buckling – The State of the Art".

2. Basic concepts of the DS bending in its buckling

Thus, further, we again shall try to demonstrate the importance of understanding the peculiarities of the DS buckling behavior inside a bore-hole and to answer the posed questions.

First of all, a deep DS is similar to human hair by conditions of their geometry and bending stiffness similarity. Then, for the sake of simplicity, consider the DS equilibrium under action of gravity forces, weight on bit and torque in vertical bore-hole. The effects of the DS rotation and mud motion are not taken into account. In this case, its critical states are described by equations (Gulyayev et al., 2009):

$$EI\frac{d^4u}{dz^4}+\frac{d}{dz}\left(T\frac{du}{dz}\right)+M_z\frac{d^3v}{dz^3}=0,\quad EI\frac{d^4v}{dz^4}+\frac{d}{dz}\left(T\frac{dv}{dz}\right)+M_z\frac{d^3u}{dz^3}=0. \tag{1}$$

Here, $u(z)$ and $v(z)$ are the lateral displacements of the DS; z is the coordinate directed along the DS axis; EI is the bending stiffness; $T(z)$ is the axial force combined by action of distributed gravity forces and weight on bit, compressing the DS at its lower end; M_z is the torque.

If length L of the DS is large, then the problem determined by Eq. (1) belongs to the so called class of singularly perturbed problems. In Chang and Howes (1984), a problem is referred to this class if the coefficients before the senior derivatives in its equations are small. In our case, coefficients EI are implicitly small. Indeed, let, for example, $L=10,000$ m. With the use of substitution $z=10,000Z$, change the scale of the z coordinate and transfer from Eq. (1) in domain $0\le z\le 10,000$ to equations

$$\frac{EI}{10^{16}}\frac{d^4u}{dZ^4}+\frac{d}{10^4dZ}\left(\frac{T}{10^4}\frac{du}{dZ}\right)+\frac{M_z}{10^{12}}\frac{d^3v}{dZ^3}=0,\quad \frac{EI}{10^{16}}\frac{d^4v}{dZ^4}+\frac{d}{10^4dZ}\left(\frac{T}{10^4}\frac{dv}{dZ}\right)+\frac{M_z}{10^{12}}\frac{d^3u}{dZ^3}=0 \tag{2}$$

in domain $0\le z\le 1$.

It can be supposed after these transformations that the first terms in Eq. (2) should be cast away, because they are very small. Then, the transformed equations acquire absolutely another structure

$$\frac{d}{dZ}\left(T\frac{du}{dZ}\right)+\frac{M_z}{10^4}\frac{d^3v}{dZ^3}=0,\quad \frac{d}{dZ}\left(T\frac{dv}{dZ}\right)+\frac{M_z}{10^4}\frac{d^3u}{dZ^3}=0 \tag{3}$$

and they become valid only for simulation of absolutely flexible strings. But flexible strings do not buckle. Therefore, if one is analyzing buckling of very long DS, he must not neglect the first terms in either Eq. (2) or Eq. (1), notwithstanding the fact that they are singularly perturbed.

As was demonstrated by Prandtl in the theory of hydrodynamic and aerodynamic flows past a body, the peculiar feature of the singularly perturbed problems is that their solutions have usually the modes of a boundary layer with unknown width. If the perturbation source acts inside the domain of the problem statement, then the solution singularity is localized in the vicinity of the perturbation (Chang and Howes, 1984). Some general ideas concerning this effect are touched by Elishakoff et al. (2001). In the problems of the DS buckling in vertical bore-holes, these resumes were corroborated in Gulyayev et al. (2009). It was shown by computer simulation that the incipient buckling of a DS in a deep vertical bore-hole takes place in its lower boundary segment and the buckling mode has the shape of a 3D irregular spiral wavelet. In Gulyayev et al. (2014), analogous conclusions were formulated relative to inclined rectilinear wells. As shown below, the buckling mode of a DS in a curvilinear bore-hole may take shape of an irregular sinusoidal wavelet localized inside the bore-hole domain. Therefore, one may not guess analytical expression of a mode of stability loss (sinusoidal or spiral), he must construct the veritable irregular mode. However, that is not easy because the most troublesome property of the singularly perturbed problems is poor calculating convergence of their solutions.

The second distinctive feature of phenomenon of a DS buckling in a curvilinear bore-hole is the extensively used assumption of ceaseless contact interaction between the DS and the bore-hole. In this situation, the bore-hole surface plays the role of a constraint and effects of critical and post-critical buckling of the DS should be considered as a 3D motion of an elastic curve in the curvilinear channel surface. Any channel surface with circular cross-section is described by complicated analytic expressions. Therefore, the considered problem is very hard but solvable, as shown below. In mechanics, two approaches based on the Lagrange methods are used for its statement. In the Lagrange method of the first type, the constraint equations are considered as additional constitutive equations and Lagrange multipliers are supposed as additional required variables. Lagrange's method of the second type is based on application of generalized coordinates. This method is more cumbersome for the problem statement but on its application the order of the constitutive equation diminishes to four, and it acquires the structure of equation of Eulerian stability of a rectilinear beam. Below, the second approach is used.

The third aspect of the problem touched is the question of the torque influence on the DS stability and modes of its buckling. If the vertical DS is not constrained by a bore-hole wall, then, as Greenhill revealed in 1883, the torque is the principal cause of its spiral buckling. This effect can be simulated with application of Eq. (1) (see also Mitchell (2008)). But if to constrain the DS displacements and exclude, for example, the $u(z)$ variable by constraint $u(z)=0$, then, the second equation will be only kept in the form

$$EI\frac{d^4v}{dz^4}+\frac{d}{dz}\left(T\frac{dv}{dz}\right)=0. \tag{4}$$

Here, M_z does not influence on the buckling process.

Moreover, below analogous conclusion is made for the DS in a curvilinear bore-hole. If the DS lies on the bottom of a plane bore-hole and only its lateral displacement is not constrained, then the torque does not influence on the buckling process (see also Gulyayev et al. (2014)). Nevertheless, when in its initial state, the DS has 3D geometry, then the torque influence should be taken into account.

The fourth issue, expecting its solution, is associated with friction forces, influencing on the buckling process. Mitchell (2008) comments: "Perhaps the most important force and the force least studied in the analysis of buckling, is friction. The magnitude of the friction force is usually not that difficult to determine. The difficulty is determining the direction of the friction vector". Furthermore, if static equilibrium of a simple frictional body system with elastic bonds is considered, the problem of the forces calculation is altogether unsoluble because it includes also a system of inequalities and has infinite number of solutions. Comprehensive analysis of this problem is performed by Wang and Yuan (2012).

But maybe, there is not a necessity to analyze stability of an immovable DS in a curvilinear bore-hole. Usually, DSs lose their stability and buckle during lowering and drilling. If to assume that