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A novel semi-analytical model for horizontal fractures with non-Darcy flow

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ABSTRACT

Based on Darcy law in matrix and non-Darcy law in horizontal fractures, a novel semi-analytical model is presented to investigate effects of non-Darcy on flow behavior. The proposed semi-analytical model convergences rapidly and it is convenient to add wellbore storage and skin effect in this model. The effects of the parameters on dimensionless pressure curves and their derivative curves are discussed in detail, including Forchheimer number, dimensionless formation thickness, fracture conductivity, and fracture diffusivity factor. Calculative results show that a big Forchheimer number, a big dimensionless formation thickness and a low conductivity will lead to big pressure depletion at early time. However, flow characteristics in entire flow regime cannot be determined by diffusivity factor. Then effects of the Forchheimer on the flow rate profiles are also discussed, indicating that as the time increases the effect of non-Darcy on flow behavior becomes weak. The proposed model could be well applied to make well test data analysis for a finite conductivity horizontal fracture with non-Darcy flow.

1. Introduction

Many studies on transient flow behavior of wells with vertical fractures have been discussed for several decades. It has been now agreed that a vertical fracture will be produced if the least principal stress in the reservoir is horizontal, while if the least principal stress is vertical a horizontal fracture will be created. The horizontal fractures have been proved and several authors have studied the steady state and transient flow behaviors. Gringarten and Ramey (1974) presented the pressure transient solutions for circular uniform-flux fractures in unbounded reservoirs by means of Green's function. Although their mathematical model can be applied to derive the pressure distribution created by a well with restricted flow entry or created by a well that only partially penetrates the formation, they did not consider the finite conductivity within the fracture or did not describe accurately the infinite conductivity behavior. Transient behavior of finite-conductivity horizontal fractures was discussed in detail by Valko and Economides (1997). In their models, a semi-analytical solution is obtained and both constant production rate and the constant drawdown cases are considered. The results could be applied to make pressure transient analysis or to forecast the production behavior. In the end of the paper, they introduced a new concept of a pseudo-skin factor for a well intercepted by a finite conductivity in horizontal fractures. Recently, Larsen (2011) presented the solutions along with examples highlighting effects of different boundary conditions option. In his paper, both compressible flow and incompressible flow in the fracture were considered. He also presented the solutions for horizontal fractures in multilayer reservoirs. All models above assumed Darcy flow within the horizontal fractures, while at present there is no literature considering the non-Darcy flow in it.

In hydraulic fractured reservoirs, the fluids usually flow from the matrix to the fracture firstly and then to the wellbore. As the fracture permeability is much higher than the reservoir permeability, so the flow rate of the fluids especially in gas reservoirs is very high, non-Darcy flow will emerge. Therefore, in this case, it is necessary to develop the models for non-Darcy flow. The flow is very complex when turbulence occurs near the wellbore. Wattenbarger and Ramey (1969) presented a finite difference model to investigate the effects of wellbore storage and turbulent flow on well test interpretation and they concluded that the effects of turbulence on early flow is strong but these effects can be ignored if the fracture is very long. Holditch and Morse (1976) studied the effects of non-Darcy flow on the behavior of hydraulically fractured gas wells. They pointed out that non-Darcy flow should be considered in the analysis of drawdown tests, pressure buildup tests, and history matching of fractured gas wells. Although they concluded that non-Darcy flow would reduce the effective fracture conductivity near the wellbore by a factor of 20 or more, they did not give any general methods of determining the actual conductivity of the fracture. Guppy et al. (1982a, 1982b) developed comprehensive numerical and semi-analytical models of non-Darcy flow to analyze the unsteady flow behavior of finite conductivity

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Nomenclature

Dimensionless variables: Real domain

C_{fD}	dimensionless fracture conductivity
F_{NDR}	Forchheimer number
r_{fD}	dimensionless max radius in the fracture
t_D	dimensionless time
r_{oD}	dimensionless radial coordinate at any point
z_{oD}	dimensionless z direction coordinate at any point
p_{wD}	dimensionless well bottom pressure
q_D	dimensionless flow rate
p_D	dimensionless pseudo-pressure
h_D	dimensionless formation thickness
dp_D	dimensionless pseudo-pressure derivative
p_{fD}	dimensionless pseudo-fracture pressure
δ	non-Darcy modifier

Dimensionless variables: Laplace domain

u	time variable in Laplace domain, dimensionless
$\overline{p}_D$	dimensionless pseudo-pressure p_D in Laplace domain
$\overline{p}_{wD}$	bottom pressure p_{wD} in Laplace domain
$\overline{p}_{fD}$	dimensionless pseudo-fracture pressure p_{fD} in Laplace domain
$\overline{q}_{fD}$	dimensionless fracture rate q_{fD} in Laplace domain

Field variables

c_{tf}	total compressibility, 1/psi
k	effective permeability, mD
k_f	fracture permeability, mD
v	velocity, m/s
β	inertial factor, 1/m
ρ	density, kg/m^3
p	bottomhole pressure, psi
q	rate of per unit fracture length from formation, MMscf/d
μ	fluid viscosity, cp
h	formation thickness, ft
ϕ_f	fracture porosity, fraction
r	reservoir radius, ft
r_f	the max radius in the fracture, ft
h	formation thickness, ft
w	width of the fracture, ft

Special functions

$K_0(x)$	modified Bessel function (2nd kind, zero order)
$K_1(x)$	modified Bessel function (2nd kind, first order)
$I_0(x)$	modified Bessel function (1st kind, zero order)
$I_1(x)$	modified Bessel function (1st kind, first order)

Special subscripts

f	fracture property
D	dimensionless
w	wellbore property

fractures. In his articles, two methods are presented to determine the true fracture conductivity when drawdown data are available at two different flow rates. The amount of turbulent effects also is quantified by means of the solutions presented. Based on Guppy et al.'s solutions, Gidley and John (1991), Roldan et al. (1996), Umnuayponwiwat and Ozkan (2000), Ramirez et al. (2007), Zeng and Zhao (2008) studied the effects of non-Darcy on the transient pressure, production performance, fracture conductivity and optimal fracture geometry in gas-drive reservoirs, dual-porosity reservoirs, dual-permeability reservoirs and naturally fractured gas condensate reservoirs.

However, all presented models assume that non-Darcy flow occurred in vertical fractured well and there is no literature referring to horizontal fractures with non-Darcy flow. The main objective of this article is to establish a semi-analytical model for horizontal fractures with non-Darcy flow firstly. Then the solution is obtained in the Laplace-transform domain and numerically inverted to the real time domain using the algorithm proposed by Stehfest (1970). In addition, the effects of non-Darcy factor on pressure transient behavior and flux distribution at different time are discussed in detail.

2. Model and algorithm

2.1. Horizontal fracture model

It is possible that non-Darcy flow occurs in horizontal fractures. The principal relationship equation for describing non-Darcy flow was proposed by Forchheimer (1901). This relationship equation can be expressed in the cylindrical system as follows:

$$\frac{dp}{dr}=\frac{\mu}{k}v+\rho\beta v^2 \tag{1}$$

Rewriting Eq. (1), the velocity equation in the horizontal fractures may be expressed as

$$v=\frac{k_f}{\mu}\delta\frac{\partial p}{\partial r} \tag{2}$$

Letting

$$\delta=\frac{1}{1+(k_f\rho\beta/2\pi\mu h)(q/r)} \tag{3}$$

We introduce dimensionless groups given as

$$q_D=\frac{q}{q_{sc}},\quad r_D=\frac{r}{r_f},\quad F_{NDR}=\frac{q_{sc}k_f\rho\beta}{2\pi r_f\mu h}$$

Eq. (3) can be rewritten in dimensionless form

$$\delta=\frac{1}{1+F_{NDR}(q_D/r_D)} \tag{4}$$

where, δ is called as the non-Darcy modifier and F_{NDR} is the Forchheimer number for non-Darcy flow in the horizontal fractures. The non-Darcy modifier δ is a function of the time. If the Forchheimer number $F_{NDR}=0$, thus $\delta=1$, then the non-Darcy flow will be simplified as Darcy flow.

It can be assumed that the horizontal fracture is represented by a disc of radius r_f, the thickness w, and inner radius of the well r_w. In the mathematical model a cylindrical system will be used with the perpendicular axis of fracture with the z axis and the center of the fracture located at $z=0$ (see Fig. 1). It follows that the fracture will be bounded by the planes $z=-w/2$ and $z=w/2$, and by the cylinders $r=r_w$ and $r=r_f$ (see Fig. 1). The fracture has permeability k_f, porosity ϕ_f, system compressibility c_{tf}, and thickness w. The horizontal fracture system is further divided into several independent segments (see Fig. 2a). Each segment of the