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# Predicting productivity index of hydraulically fractured formations

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ABSTRACT

Productivity index and inflow performance of horizontal wells intersecting multiple hydraulic fractures are of great importance. This importance comes from the fact that the fracturing process has become a common stimulation technique in the petroleum industry. However, few models for the productivity index and inflow performance have been presented in the literatures due to the complexity governing this topic.

This paper introduces a new technique for estimating the pseudo-steady state productivity index of horizontal wells intersecting multiple hydraulic fractures. Based on the instantaneous source solutions for the diffusivity equation, seven analytical models have been derived for different source solutions. Four of them represent the effect of the formation height and fracture height (the vertical direction), while the other three represent the solution for the horizontal plane. For vertical hydraulic fractures, the four solutions of the vertical direction, representing the pseudo-skin factor, are almost neglected. The three horizontal plane solutions are the main parameters that control the productivity index and inflow performance of the fractured formations. In this technique, the horizontal wells are acting in finite reservoirs where the pseudo-steady state flow is expected to develop. Reservoir geometry, reservoir properties, and fracture dimensions were considered in this technique. The number of fractures and the spacing between them were also investigated in this study. A new analytical model for estimating the required number of hydraulic fractures has been introduced in this study based on the reservoir drainage area and the surface area of fractures.

The models have been used to establish several plots to estimate the shape factor group based on the number of fractures and the half fracture length. This group is one of the main terms in the productivity index model. Several plots for the shape factor of fractured formations have been introduced in this study. The results obtained from the new technique have been compared with the results from previous models. Several numerical examples will be included in the paper.



## 1. Introduction

Hydraulic fracturing is an important stimulation technique that has been widely used in conventional and unconventional oil and gas reservoirs all over the world. The technique involves the creation of a fracture or the fracture system in porous medium to (1) increase the contact area between reservoir matrix and the wellbore, (2) overcome wellbore damage, (3) improve oil and gas productivity in low permeability and tight gas reservoirs, (4) enhance the connectivity of naturally fractured reservoirs, and (5) facilitate the production from shale gas reservoirs. During the last two decades, horizontal wells with multi-stage hydraulic fractures have become a common applied completion technology in the petroleum industry. Because of the large reservoir contact area connected to the wellbore, hydraulic fractures can greatly improve a well's productivity.

The productivity index is a critical parameter in the oil and gas production process and its management. Regardless of the type of formation and the type of wellbore, the index is defined as the amount or volume of reservoir fluids that can be produced daily by 1 psi pressure drop at the sand face. For a horizontal well with multiple hydraulic fractures, the productivity index is influenced by several factors such as the number of fracture, the spacing between them and the fracture dimensions. Reservoir permeability and reservoir fluid properties have great influence on the productivity index as well as the geometry of the drainage area. Several models have been introduced during the last two decades for the productivity index of fractured formations. The high cost of the fracturing process and the serious need for a technique that can help in evaluating the benefits of the continuously increasing number of fractures are the two motivating factors to develop the productivity index models for fractured formations.

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**Nomenclature**

| | |
|---|---|
| $B$ | formation volume factor, res-bbl/STB |
| $C_A$ | shape factor, dimensionless |
| $C_{HF}$ | shape factor group, dimensionless |
| $h$ | formation height, ft |
| $h_f$ | fracture height, ft |
| $c_t$ | total compressibility, psi$^{-1}$ |
| $J$ | productivity index, dimensionless |
| $Jp$ | pseudo-productivity index, dimensionless |
| $k_x$ | permeability in the X-direction, md |
| $k_y$ | permeability in the Y-direction, md |
| $k_z$ | permeability in the Z-direction, md |
| $N$ | number of fractures |
| $\Delta P$ | pressure difference, psi |
| $Q$ | total flow rate, STB/D |
| $q$ | fracture flow rate, STB/D |
| $rf$ | equivalent wellbore radius |
| $s$ | spacing between fractures, ft |
| Sp | pseudo-skin factor, dimensionless |
| Sm | mechanical skin factor, dimensionless |
| $t$ | time, h |
| $x_e$ | reservoir half width, ft |
| $x_f$ | fracture half length, ft |
| $y_e$ | reservoir half length, ft |
| $\phi$ | porosity |
| $\mu$ | viscosity, cp |
| $\eta$ | diffusivity |

Raghavan and Joshi (1993) presented a method to evaluate the productivity of wells with hydraulic fractures. A general and rigorous model was provided in their study for the productivity index as a function of reservoir variables and numbers of radials of fractures. Guo and Evans (1993) proposed a new analytical model for the productivity index of fractured formations. They stated that the transient flow period cannot be used to estimate the productivity index since the drainage area is variable during oil and gas production. Therefore the dynamic productivity index based on the uniform fracture flow rate distribution would be used for this purpose. Their most important finding was the negligible impact of the fracture half length on the productivity index during the pseudo-steady state. Li et al. (1996) developed a model to predict the performance of fractured formations and examined it for two case studies. They investigated the effects of the number of fractures and fracture half length on the productivity of the wells.

Valko and Economides (1998) suggested the use “proppant number,” proportional to the fracture permeability and propped volume and conversely proportional to the reservoir permeability and reservoir volume. This number can be used to estimate the productivity index corresponding to the fracture conductivity in pseudo-steady state flow. Larsen (1998, 2001) introduced several analytical models for the productivity of fractured and non-fractured deviated wells in commingled reservoirs. Fokker et al. (2005) presented a novel approach to determine the productivity of complex wells. They stated that their model is applicable for the finite-conductivity wells, well interference, non-homogenous reservoirs, and hydraulically fractured formations. Sadrpanah et al. (2006) studied the impact of the fracturing process on the deliverability of a well. Their study has been conducted on a pilot plant where two fracture systems and four fractures were used to enhance the production.

The performance of the horizontal wells associated with multiple hydraulic fractures depends on the penetration ratio in two directions. The first one is the vertical direction where the height of fracture does not need to be equal to the height of the formation. The second is the horizontal direction where the fracture wings are designed to propagate; however it is not common that the fracture tips reach the boundaries. Demarchos et al. (2006) investigated the effect of the drainage shape and flow regimes in transversely fractured wells on the productivity. The concept of “proppant number,” developed by Valko and Economides, has been also used by Demarchoes et al. Guo et al. (2006) presented a new model to predict fractured horizontal wells production. The model combined the shape of formed fractures and the fluid peculation mechanism together.

The productivity and the drainage area of fractured horizontal wells in tight gas reservoir and shale plays are of great importance in the petroleum industry. The importance comes from the high cost of both the completion techniques for these wells and the fracturing process that may include more than 20 stages. Medeiros et al. (2008) discussed the performance of the fractured horizontal wells in heterogeneous and tight gas formations. They documented in their study the production characteristics and flow regimes in which the long transient periods may govern the productivity. Guo and Yu (2008) stated that the horizontal wells drilled in the direction of the minimum horizontal stress allow multiple transverse hydraulic fractures to enhance well productivity in low permeability oil and gas reservoirs. Two case studies have been investigated in their argument using three analytical models for three expected flow regimes. Grieser et al. (2009) used a 3-D four phase nonisothermal multiwall black oil and pseudo-compositional simulator that allows placement of multiple transverse fractures to predict production outcome from horizontal completions in the Barnett Shale. Zhang et al. (2012) approached a method to predict the total inflow performance and associated productivity index under two-phase flow conditions. The approach can be used to determine the number of fracture stages required for horizontal wells extending in a two-phase solution-gas drive.

## 2. Mathematical modeling

The hydraulic fracturing process aims to greatly increase the contact area between the wellbores and the pay zones. The production rate from fractured formations is a function of the total surface area of the hydraulic fractures. For a constant production rate, the pressure drop at any point in fractured formations depends on several parameters: permeability, homogeneity, isotropy, formation drainage area configuration, reservoir fluid properties, and fracture dimensions. Total production rates from all hydraulic fractures and total pressure drop at the wellbore are the two items required for estimating the productivity index. The worthy and successful fracturing process has to significantly improve the productivity index of the formation. Therefore, the productivity index is a critical parameter in oil and gas production from fractured formation because of the high cost of the single stage fracturing process. Generally, the simple model for the productivity index can be written as

$$J = \frac{q}{\Delta P} \tag{1}$$

For constant sandface production rate, the pressure drop is the main parameter that affects the productivity index. Pressure drop