Permeability from porosimetry measurements: Derivation for a tortuous and fractal tubular bundle

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A B S T R A C T

Permeability modeling of complex carbonate reservoirs is difficult. Porosity–permeability relationships are weak in carbonates and conventional porosity–permeability transforms give poor results. Even though the link between porosity and permeability in carbonates persists, other pore system properties, such as the largest connected pore-throat diameter, are more strongly linked to permeability. This important pore-throat diameter, as well as related porosity and other pore system architectural information, is determined by the analysis of mercury injection capillary pressure (MICP) porosimetry experiments. This paper explores the use of porosimetry data for the calculation of permeability as originally demonstrated by Purcell in 1949. We return to the tubular bundle model of Purcell and Burdine with a general mathematical form for the porosimetry data and a new tortuous and fractal relative tubular bundle. Using mathematical reasoning, without fitting parameters, we obtain a new formula for the computation of permeability based on the pore system architectural information of highly connected systems using the MICP porosimetry data. Moreover, we include the observation that the flow paths and the related tortuosity have a fractal aspect. The result is compared to an extensive porosimetry data set of the highly connected Arab D limestone, where vugs are absent. For porous media characterized by porosimetry data, the following expression emerges:

$$\kappa \approx 506 \frac{B_v^\infty}{P_d^2} e^{-4.43\sqrt{G}},$$

which is the permeability for a monomodal carbonate pore system characterized by a single Thomeer hyperbola with associated Thomeer parameters (κ is in Darcy; B_v^∞ in fractional bulk volume, and P_d is the minimum entry pressure in psi and G is the pore-geometrical factor). The nearly equal sign is used here only because of an approximation used for the modified Bessel function of the second kind present in the general solution and approximate knowledge of the fractal exponent and the percolation path length ratio. There are no fitting factors. The exponents on the variables in our permeability formula demonstrate the significant shift in emphasis from porosity to the diameter of the largest connected pore throats, P_d. Note that the presence of vugs are not considered in this work, since they do not form part of the Arab-D limestone matrix.

This mathematical effort emphasizes the relative importance of pore system attributes on permeability as commonly found in carbonate porosimetry data. The approach can be readily extended to multimodal carbonate pore systems, to other sources of pore system architectural data and is shown to be equivalent to the operation of an incomplete Laplace transform on the porosimetry data. Importantly, and in contrast to previous permeability models to which we compare, this new formulation sets the stage for a complete and scale independent understanding of permeability in carbonate pore systems commonly encountered in the Arab D limestone and similar pore systems.

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1. Introduction

The carbonate oil reservoirs of Saudi Arabia are the most prolific in the world. Their porosities and permeabilities are excellent, but detailed understanding of the petrophysical properties of these complex carbonate pore systems is insufficient. The carbonate pore systems are complex and multimodal (Cantrell and Hagerty, 1999; Cantrell and Hagerty, 2003; Clerke et al., 2008) leading to poor porosity–permeability relationships. Standard porosity–permeability transforms have major and well-known shortcomings (Delfiner, 2007) especially in the case of carbonates. Estimating accurate matrix permeability for the Arab-D limestone

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from porosity alone is unsatisfactory (Clerke et al., 2008; Clerke, 2009). Additional information about the pore systems is needed. Saudi Aramco has acquired many mercury injection capillary pressure (MICP) porosimetry data on Arab D limestone core plugs (Clerke et al., 2008), which are analyzed using the method proposed by Thomeer (1960) who observed a hyperbolic relationship between the fractional bulk volume of injected mercury, log (B_v) and the applied mercury pressure, $\log(P_c)$. The amount of mercury entering a single pore system in an MICP experiment, expressed as the fractional bulk volume occupied B_v, exhibits a distinct hyperbolic shape. Thomeer's observation is represented by his empirical formula:

$$B_v(P_c) \approx B_v^{\infty} \exp\left(\frac{-G}{\log(P_c) - \log(P_d)}\right) \quad \text{for } P_c > P_d; \; 0 \text{ elsewhere}. \quad (1)$$

The details in position and shape of the hyperbola are determined by the three Thomeer parameters: P_d, which is the "minimum entry pressure" or "threshold pressure", related to the largest connected pore throat diameter; G is the "pore geometrical factor" related to the range of pore throat diameters and B_v^{∞}, which is the "fractional bulk volume occupied" by mercury at $P_c = \infty$. P_c is the applied mercury pressure, normally in psi.

These three pore system parameters determine the pore geometrical capillary forces within the reservoir pore system and exert important control on the original distribution of water and oil within the reservoir pore system. This significant pore geometrical information must also contain the information regarding the flow of liquids through the reservoir and thus largely define absolute permeability. This realization was introduced by Purcell (1949) and Burdine (1953) and later successfully developed by Thomeer (1983) and Swanson (1981), and others (Huet et al., 2005), despite the fact that pseudo-static MICP data do not give explicit information on the accessibility, complexity and tortuosity of the flow paths. Our efforts expand upon that of previous workers by using rigorous mathematics and by introducing a tortuous and fractal relative tubular bundle. The result is a completely new functional form for permeability. We proceed to compare our new formulation to abundant Arab D carbonate MICP data (Clerke et al., 2008).

2. Permeability of a tortuous tubular bundle

We represent the actual pore system by an assortment of different tubes, connecting one end of the rock to the other, starting with the approach of Purcell (1949), Burdine (1953), and Calhoun et al. (1949). In Appendix A, a step-by-step derivation is given. It leads ultimately to the general equation, which will be summarized in this section.

The total volume of liquid passing through a piece of rock of length, L, and cross-sectional area, A, is described by the standard Muscat–Darcy fluid-flow equation as

$$\frac{dV_{liq}}{dt} = \frac{\kappa A}{\mu L} \Delta P, \quad (2)$$

where μ is the viscosity of the liquid and ΔP is the pressure drop over the length of the tube. κ is the permeability which needs to be expressed in terms of fundamental rock properties, such as porosity and pore architectural parameters.

Under a pressure gradient, liquids flow through a porous rock along certain paths, which are envisaged as well-defined pipes or tubes. We assume that the flow is non-turbulent. In this tubular bundle representation of a rock pore system, the tubes connect one side of the rock sample with the other. It is assumed that the tubes have varying dimensions, different radii, r_i, and different lengths L_i. Moreover, the tubes are tortuous and not straight, so

that the length of a tube is longer than the sample outer length, i.e. $L_i > L$.

The tubes are identified by their radii, r_i. Within the "i-family", there are n_i tubes. Each tube of radius r_i can in principle have a different length. However, here, we assume that they all have the same length L_i, where L_i is an average length of all the tubes with radius r_i.

The lengths of the tubes are also modeled to increase with decreasing diameters, i.e. $L_{i+1} > L_i$ (throughout this paper we use the convention that the radius decreases with increasing index i, i.e. $r_{i+1} < r_i$). This is reasonable because the flow paths through the narrower passages of the pore system are expected to be more tortuous and thus longer. Moreover these tubes of narrow diameters should be more numerous, i.e. $n_{i+1} > n_i$.

The Hagen–Poiseuille flow equation for this bundle of tubes can be written as (for details see Appendix A):

$$\frac{dV_{liq}}{dt} = \sum_i n_i \frac{\pi r_i^4}{8\mu L_i} \Delta P. \quad (3)$$

From Eqs. (2) and (3), the permeability is extracted as

$$\kappa = \frac{1}{8} \sum_i n_i \frac{\pi r_i^4}{A} \frac{L}{L_i}. \quad (4)$$

The volume of a single tube with radius r_i is $v_i = \pi r_i^2 L_i$ and the volume of all the tubes with radius r_i is $V_i = n_i v_i = n_i \pi r_i^2 L_i$. The volume of the bulk rock $V_b = AL$. With these, we can rewrite (4) as

$$\kappa = \frac{1}{8} \sum_i \underbrace{\frac{n_i \pi r_i^2 L_i}{AL}}_{\Delta\varphi_i} \left(\frac{L}{L_i}\right)^2 r_i^2 = \frac{1}{8} \sum_i \Delta\varphi_i r_i^2 \left(\frac{L}{L_i}\right)^2, \quad (5)$$

where the incremental porosity is introduced, $\Delta\varphi_i = V_i/V_b$, which is related to all the tubes with radius r_i.

The total porosity is the sum of all incremental porosities: $\phi = \sum_i \Delta\varphi_i$, and porosity normalization yields

$$\kappa = \frac{1}{8} \phi \sum_i \frac{\Delta\varphi_i}{\phi} r_i^2 \left(\frac{L}{L_i}\right)^2. \quad (6)$$

Now, we denote the properties of the largest diameter tube with the special subscript, "d". So the largest diameter tube has a radius of r_d and a length of L_d. The subscript "d" anticipates the relationship with P_d, the minimum entry pressure for mercury (threshold pressure) where in an MICP experiment the mercury first enters the largest pore throat by penetrating into the throat (s) of radius r_d.

It is now convenient to denote the dimensions of all other tubes, i.e. radii and lengths, relative to that of the largest tube in the system, by rewriting the previous equation as (see Eq. (A4))

$$\kappa = \frac{1}{8} \phi \sum_i \frac{\Delta\varphi_i}{\phi} r_i^2 \left(\frac{L}{L_i}\right)^2 \left(\frac{r_d}{r_d}\right)^2 \left(\frac{L_d}{L_d}\right)^2 \Leftrightarrow \kappa = \frac{1}{8} r_d^2 \phi \left(\frac{L}{L_d}\right)^2 \sum_i \frac{\Delta\varphi_i}{\phi} \left(\frac{r_i}{r_d}\right)^2 \left(\frac{L_d}{L_i}\right)^2 \quad (7)$$

The contributions of the tube radii and lengths are now all referenced to the radius and length of the biggest tube in the system. It is reasonable to assume that the biggest tube has also the shortest length L_d, which is smaller than L_i. Of course L_d is larger than the length L of the sample, so that

$$0 \leq \frac{L_d}{L_i} \leq 1 \quad \text{and} \quad 0 \leq \frac{r_i}{r_d} \leq 1$$

Let χ_L be the relative tube parameter product defined as

$$\chi_L^2 \equiv \left(\frac{r_i}{r_d}\right)^2 \left(\frac{L_d}{L_i}\right)^2 < 1. \quad (8)$$

Then since $\sum_i \Delta\varphi_i/\phi = 1$, the porosity weighted square relative tube parameter is $\sum_i \Delta\varphi_i/\phi \chi_L^2 = \langle \chi_L^2 \rangle$ and Eq. (7) can now be written