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Effects of suction/injection on unsteady reactive variable viscosity non-Newtonian fluid flow in a channel filled with porous medium and convective boundary conditions

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ABSTRACT

A study on thermal effects of the suction/injection Reynolds number, in conjunction with other flow parameters, on an unsteady reactive temperature dependent viscosity third grade fluid in a porous channel filled with saturated porous medium is presented. It is assumed that the channel walls are subjected to asymmetric convective heat exchange with the surrounding medium and that exothermic chemical reactions take place within the flow system. The heat exchange with the ambient at the surfaces is assumed to obey Newton's law of cooling. The equations governing the flow system are expressed in non-dimensional form and a semi-implicit finite difference scheme is utilised to obtain the velocity and temperature profiles. The effects of the flow parameters on the temperature and velocity fields, the skin friction and the wall heat transfer rate are simulated and discussed. The suction/injection Reynolds number is observed to retard the velocity field. It is also observed that the suction/injection Reynolds number, the porous medium parameter, the Prandtl number and the Biot number have a retarding effect on the temperature field. The variable viscosity parameter and the suction injection Reynolds number increases the skin friction while the porous medium parameter and the Prandtl number diminish it. It is also revealed that the suction injection Reynolds number, the porous medium parameter and the Prandtl number have a diminishing effect on the rate of heat transfer at the channel walls.

1. Introduction

The flow of viscous fluids in channels with porous walls filled with or without porous media has been investigated and studied by several scholars for various physical situations. In such flow systems, consideration of thermal effects and thermal stability criteria ought to be a major part of the analysis. Diverse applications are found in geothermal energy extraction, drying of food, nuclear waste disposal, heat and fluid exchange inside human organs, insulation of buildings, groundwater movement, oil and gas production, surface catalysis of chemical reactions, regenerative heat exchange and adsorption, etc. In some of these physical systems, the fluids involved belong to the wide class of non-Newtonian fluids owing to their failure in obeying the classical linear viscosity model. The heuristics and mathematical computations underlying the rheology of such fluids, in particular fluids of the differential type, are cumbersome and complex as to evoke the interest of many scholars. In an incompressible fluid of differential type, apart from a constitutively indeterminate pressure, the stress is just a function of the velocity gradient and a number of its higher time derivatives (Fosdick and Rajagopal, 1980).

Beg and Makinde (2011) presented a theoretical analysis of a two-dimensional steady, laminar flow of an incompressible, viscous elastic fluid with species diffusion in a parallel plate channel with porous walls containing a homogeneous, isotropic porous medium with high permeability. The Runge–Kutta integration scheme with a modified version of the Newton–Raphson shooting method imbedded in the MAPLE software was used to numerically solve the transformed similarity of the ordinary differential equations governing the system. Makinde and Chinyoka (2010, 2012) analysed unsteady flow of a variable viscosity reactive fluid and heat transfer in a circular pipe with porous walls and in a slit with wall suction or injection respectively. Makinde and Ogulu (2008) investigated the effects of thermal radiation on the heat and mass transfer flow of a variable viscosity fluid past a vertical porous plate permeated by a transverse magnetic field. Wang et al. (2001) conducted a theoretical investigation of the flow distribution and pressure drop in a channel with porous walls. Attia (2005) studied the effect of suction and injection on the unsteady flow between parallel plates with variable viscosity and thermal conductivity.

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The effect of suction and injection on magnetohydrodynamic three dimensional couette flow and heat transfer through a porous medium was studied by Das (2009). Other related studies are found in Makinde (1996, 2006, 2007) and Fang (2004).

In this study we seek to conduct an in-depth succinct analysis whereby we outline longitudinal velocity and temperature profiles of an unsteady reactive temperature dependent viscosity third grade fluid flow between two permeable parallel plates filled with a saturated porous medium. The dependence of the velocity and temperature fields, the skin friction and the Nusselt number on the many flow parameters, in particular the suction/injection Reynolds number, is extensively analysed. The ultimate aim is to characterise thermal effects and thermal stability criteria of the flow regime as dictated by the underlying parameters. Such detailed thermodynamics investigation has far reaching implications on safety in the industries dealing with flow systems of this nature.

The organisation of the rest of the paper is as follows: The physical problem is presented and formulated mathematically in Section 2. The solution process, a semi-implicit finite difference scheme based on Chinyoka (2008, 2011), is implemented in Section 3. Section 4 comprises the simulations of the velocity and temperature profiles together with the discussion of the results.

2. Mathematical formulation

We examine an unsteady flow of an incompressible variable viscosity, reactive non-Newtonian fluid through a channel filled with a homogeneous and isotropic porous medium. It is assumed that the channel walls are uniformly porous such that fluid injection and suction take place at the lower and upper walls respectively as illustrated in Fig. 1. The plate surfaces are also subjected to asymmetric convective heat exchange with the ambient due to unequal heat transfer coefficients and the fluid motion is induced by an applied axial pressure gradient. We choose the $\vec{x}$-axis parallel to the channel and the $\bar{y}$-axis normal to it.

Following Brinkman (1947), Truesdell and Noll (1965), Rajagopal (1995), Fosdick and Rajagopal (1980), Som et al. (2005), Frank-Kamenetskii (1969) and Al-Hadhrami et al. (2003), and neglecting the reacting viscous fluid consumption, the governing equations for the momentum and heat balance can be written as

$$\rho\left(\frac{\partial u}{\partial \bar{t}}+V\frac{\partial u}{\partial \bar{y}}\right)=\frac{\partial \overline{P}}{\partial \bar{x}}+\frac{\partial}{\partial \bar{y}}\left[\bar{\mu}(T)\frac{\partial u}{\partial \bar{y}}\right]+\alpha_1\frac{\partial^3 u}{\partial \bar{y}^2\partial \bar{t}}+6\beta_3\frac{\partial^2 u}{\partial \bar{y}^2}\left(\frac{\partial u}{\partial \bar{y}}\right)^2-\frac{\bar{\mu}(T)u}{\rho K}, \tag{1}$$

$$\rho c_p\left(\frac{\partial T}{\partial \bar{t}}+V\frac{\partial T}{\partial \bar{y}}\right)=k\frac{\partial^2 T}{\partial \bar{y}^2}+\left(\frac{\partial u}{\partial \bar{y}}\right)^2\left(\bar{\mu}(T)+2\beta_3\left(\frac{\partial u}{\partial \bar{y}}\right)^2\right)+\frac{\bar{\mu}(T)u^2}{K}+QC_0A\left(\frac{hT}{\nu l}\right)^m e^{-E/RT}. \tag{2}$$

The additional viscous dissipation term in Eq. (2) is valid in the limit of very small and very large porous medium permeability.

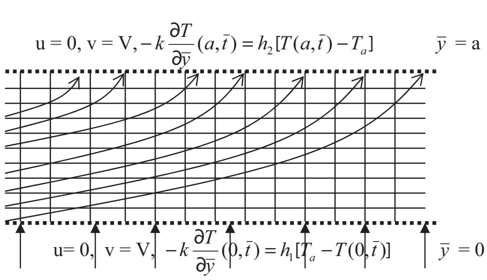

Fig. 1. Schematic diagram of the problem.

The appropriate initial and boundary conditions are

$$u(\bar{y},0)=0,\quad T(\bar{y},0)=T_0, \tag{3}$$

$$u(0,\bar{t})=0,\quad -k\frac{\partial T}{\partial \bar{y}}(0,\bar{t})=h_1\left[T_a-T(0,\bar{t})\right], \tag{4}$$

$$u(a,\bar{t})=0,\quad -k\frac{\partial T}{\partial \bar{y}}(a,\bar{t})=h_2\left[T(a,\bar{t})-T_a\right]. \tag{5}$$

Here T is the absolute temperature, ρ is the density, c_p is the specific heat at constant pressure, $\bar{t}$ is the time, h_1 is the heat transfer coefficient at the lower plate, h_2 is the heat transfer coefficient at the upper plate, T_0 is the fluid initial temperature, T_a is the ambient temperature, k is the thermal conductivity of the material, Q is the heat of reaction, A is the rate constant, E is the activation energy, R is the universal gas constant, C_0 is the initial concentration of the reactant species, $a$is the channel width, l is Planck's number, h is Boltzmann's constant, $\nu$is the vibration frequency, K is the porous medium permeability, α_1 and β_3 are the material coefficients, $\overline{P}$ is the modified pressure, and m is the numerical exponent such that $m\in\{-2,0,0.5\}$, where the three values represent numerical exponents for sensitised, Arrhenius and bimolecular kinetics respectively (see Makinde (2006, 2009) and Frank-Kamenetskii (1969)). The temperature dependent viscosity ($\bar{\mu}$) can be expressed as

$$\bar{\mu}(T)=\mu_0 e^{-b(T-T_0)}, \tag{6}$$

where b is a viscosity variation parameter and μ_0 is the initial fluid dynamic viscosity at temperature T_0. We introduce the following dimensionless variables into Eqs. (1)–(6)

$$y=\frac{\bar{y}}{a},\ \alpha=\frac{bRT_0^2}{E},\ w=\frac{u\rho a}{\mu_0},\ \theta=\frac{E(T-T_0)}{RT_0^2},\ \theta_a=\frac{E(T_a-T_0)}{RT_0^2},$$
$$Re=\frac{Va\rho}{\mu_0},\gamma=\frac{\beta_3\mu_0}{\rho^2a^4},\ \delta=\frac{\alpha_1}{\rho a^2},\ Bi_1=\frac{h_1a}{k},\ Bi_2=\frac{h_2a}{k},$$
$$Da=\frac{K}{a^2},\ Pr=\frac{\mu_0c_p}{k},\varepsilon=\frac{RT_0}{E},\ x=\frac{\bar{x}}{a},\ P=\frac{\overline{P}\rho a^2}{\mu_0^2},$$
$$G=-\frac{\partial\overline{P}}{\partial\bar{x}},\ S^2=\frac{1}{Da},\ t=\frac{\bar{t}\mu_0}{\rho a^2},\mu=\frac{\bar{\mu}}{\mu_0},\ \lambda=\left(\frac{hT_0}{\nu l}\right)^m\frac{QEAa^2C_0e^{-E/RT}}{T_0^2Rk},$$
$$\Omega=\left(\frac{\nu l}{hT_0}\right)^m\frac{\mu_0^3e^{-E/RT}}{\rho^2QAa^4C_0}, \tag{7}$$

and obtain the following dimensionless governing equations:

$$\frac{\partial w}{\partial t}+Re\frac{\partial w}{\partial y}=G-S^2we^{-\alpha\theta}+e^{-\alpha\theta}\frac{\partial^2 w}{\partial y^2}-\alpha e^{-\alpha\theta}\frac{\partial\theta}{\partial y}\frac{\partial w}{\partial y}+\delta\frac{\partial^3 w}{\partial y^2\partial t}+6\gamma\frac{\partial^2 w}{\partial y^2}\left(\frac{\partial w}{\partial y}\right)^2, \tag{8}$$

$$Pr\frac{\partial\theta}{\partial t}+PrRe\frac{\partial\theta}{\partial y}=\frac{\partial^2\theta}{\partial y^2}+\lambda\left\{(1+\varepsilon\theta)^m\exp\left(\frac{\theta}{1+\varepsilon\theta}\right)+\Omega\left[S^2w^2e^{-\alpha\theta}+\left(\frac{\partial w}{\partial y}\right)^2\left(e^{-\alpha\theta}+2\gamma\left(\frac{\partial w}{\partial y}\right)^2\right)\right]\right\} \tag{9}$$

$$w(y,0)=0,\quad \theta(y,0)=0, \tag{10}$$

$$w(0,t)=0,\quad \frac{\partial\theta}{\partial y}(0,t)=-Bi_1[\theta_a-\theta(0,t)], \tag{11}$$

$$w(1,t)=0,\quad \frac{\partial\theta}{\partial y}(1,t)=-Bi_2[\theta(1,t)-\theta_a], \tag{12}$$

where λ represents the Frank-Kamenetskii parameter, Pr is the Prandtl number, Bi is the Biot number, ε is the activation energy parameter, δ is the material parameter, γ is the non-Newtonian

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