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Finite element modelling of viscosity-dominated hydraulic fractures

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ABSTRACT

Hydraulic fracturing is a highly effective technology used to stimulate fluid production from reservoirs. The fully 3-D numerical simulation of the hydraulic fracturing process is of great importance to developing more efficient application of this technology, and also presents a significant technical challenge because of the strong nonlinear coupling between the viscous flow of fluid and fracture propagation. By taking advantage of a cohesive zone method to simulate the fracture process, a finite element model based on existing pore pressure cohesive finite elements has been established to simulate the propagation of a viscosity-dominated hydraulic fracture in an infinite, impermeable elastic medium. Selected results of the finite element modelling and comparisons with analytical solutions are presented for viscosity-dominated plane strain and penny-shaped hydraulic fractures, respectively. Some important issues such as mesh transition and far-field boundary approximation in the cohesive finite element model have been investigated. Excellent agreement between the finite element results and analytical solutions for the limiting case where the fracture process is dominated by fluid viscosity demonstrates the capability of the cohesive zone finite element model in simulating the hydraulic fracture growth.

1. Introduction

Hydraulic fracturing is a powerful technology mainly used in the petroleum industry to stimulate reservoirs to enhance oil and/or gas production. Other important and successful applications include determination of in situ stress in rock (Haimson and Fairhurst, 1970), preconditioning rock for caving or inducing rock to cave in mining (van As and Jeffrey, 2000; Jeffrey et al., 2001), creation of geothermal energy reservoirs, and underground disposal of toxic or radioactive waste (Sun, 1969). The recent global fast growing development of unconventional gas also requires novel methods of hydraulic fracturing. Furthermore, natural hydraulic fractures are manifest as kilometre-long volcanic dykes that bring magma from deep underground chambers to the earth's surface, or as sub-horizontal fractures known as sills diverting magma from dykes (Spence and Turcotte, 1985; Lister and Kerr, 1991; Rubin, 1995).

During a standard industrial treatment, the appropriate amounts of fracturing fluid and proppant are blended and pumped into the rock mass at high enough injection rates and pressures to open and extend a fracture hydraulically. Minimising the energy required for propagation dictates that the hydraulic fracture tends to develop in a direction perpendicular to the direction of the minimum principal in situ compressive stress. Typically hydraulic fracturing involves four important coupling processes (Bunger et al., 2005; Adachi et al., 2007): (i) the rock deformation induced by the fluid pressure on the fracture faces; (ii) the flow of viscous fluid within the fracture; (iii) the fracture propagation in rock; and (iv) the leak-off of fluid from the fracture into the rock formation. Therefore, fully modelling the hydraulic fracturing process requires solving a coupled system of governing equations consisting of (Khristianovic and Zheltov, 1955; Spence and Sharp, 1985; Clifton, 1989; Detournay, 2004; Bunger et al., 2005; Bunger and Detournay, 2008) (1) elasticity equations that determine the relationship between the fracture opening and the fluid pressure, (2) non-linear partial differential equations for fluid flow (usually obtained from lubrication theory) that relate the fluid flow in the fracture to the fracture opening and the fluid pressure gradient, (3) a fracture propagation criterion (usually given by assuming linear elastic fracture mechanics is valid) that allows for quasi-static fracture growth when the stress intensity factor is equal to the rock toughness, and (4) diffusion of fracturing fluid into the rock formation.

The problem associated with modelling hydraulic fractures has been addressed by a large number of papers, starting with the pioneering work by Khristianovic and Zheltov (1955). The early research efforts concentrated on obtaining analytical solutions for the complex fluid–solid interaction problems by assuming a simple fracture geometry, resulting in the well-known 2-D plane strain PKN and KGD models, and the axisymmetric penny-shaped model (Perkins and Kern, 1961; Geertsma and de Klerk, 1969; Sun, 1969; Abe et al., 1976). These approaches typically rely on simplification of the problem either with respect to the fracture opening profile or the fluid pressure distribution. Because of the geometric limitations of analytical models, a good deal

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of effort has been applied to the development of numerical models to simulate the propagation of hydraulic fractures for more complex and realistic geometries, with the first such so-called pseudo-3D model developed in the late 1970s (Settari and Cleary, 1984). Significant progress has been made in developing 2-D and 3-D numerical hydraulic fracture models (Vandamme and Curran, 1989; Zhang et al., 2002, 2007; Zhang and Jeffrey, 2006; Adachi et al., 2007; Lecamplon and Detournay, 2007; Dean and Schmidt, 2009; Ji et al., 2009; Zhang and Ghassemi, 2011). In recent years, some newly developed numerical methods, such as the extended finite element method (XFEM), have been applied to investigating hydraulic fracture problems (Lecampion, 2009). However, because of the difficulty posed by modelling a fully 3-D hydraulic fracture, numerical simulation still remains a particularly challenging problem (Peirce and Detournay, 2008).

The cohesive zone finite element method, which has its origin in the concepts of a cohesive zone model for fracture originally proposed by Barenblatt (1962) and Dugdale (1960), has been extensively used with great success to simulate fracture and fragmentation processes in concrete, rock, ceramics, metals, polymers, and their composites. Rather than an elastic crack tip region as presumed in classic linear elastic fracture mechanics with its associated infinite stress at the crack tip, the cohesive zone model assumes the existence of a simplified fracture process zone characterised by a traction-separation law. In this way, the cohesive zone model avoids the singularity in the crack tip stress field that is present in classic fracture mechanics. In addition, the cohesive zone model fits naturally into the conventional finite element method, and thus can be easily implemented. So the cohesive finite element method provides an alternate, effective approach for quantitative analysis of fracture behaviour through explicit simulation of the fracture processes.

Compared to the conventional fracture mechanics method, the cohesive element method has the following advantages in modelling hydraulic fracturing. Firstly, the cohesive zone model effectively avoids the singularity at the crack tip region, which poses considerable challenges for numerical modelling in classic fracture mechanics. The lubrication equation, governing the flow of viscous fluid in the fracture, involves a degenerate non-linear partial differential equation (Peirce and Detournay, 2008). The coefficients (permeability) in the principal part of this equation vanish as a power of the unknown fracture width (opening). The fracture opening tends to zero near the tip of an elastic crack as described by classic fracture mechanics. This non-linear degeneracy poses a considerable challenge for numerical modelling. While, in a cohesive zone model, fracture opening is not zero but finite at the cohesive crack tip, which naturally avoids the non-linear degeneracy problem associated with the singularity in fluid pressure that otherwise must be handled at the crack tip. Secondly, the hydraulic fracture propagation is a moving boundary value problem in which the unknown footprint of the fracture and its encompassing boundary need to be found while specifying an additional fracture propagation criterion in the classic fracture mechanics method. While in the cohesive zone finite element model, the location of the crack tip is not an input parameter but a natural, direct outcome of the solution, which increases the computation efficiency. In addition, the cohesive zone model has the interesting capability of modelling microstructural damage mechanisms inherent in hydraulic fracturing such as initiation of micro cracking and coalescence, and the initiation of a hydraulic fracture from a borehole. Sarris and Papanastasiou (2011) investigated the influence of cohesive process zone in hydraulic fracture modelling. Chen et al. (2009) have applied the cohesive element method to modelling a toughness dominated penny-shaped hydraulic fracture. In this paper, the cohesive element method has been used to simulate the propagation of a hydraulic fracture in viscosity-dominated regime. An innovative meshing scheme using mesh transition and node sharing techniques has been applied in the simulation, which provides a high solution accuracy and efficiency. In addition, the use of the analytical solution of an equivalent displacement discontinuity singularity provides an accurate description of the far-field boundary conditions for modelling fractures embedded in an infinite domain, and is computationally efficient.

2. Cohesive model of hydraulic fracture

As illustrated in Fig. 1, a fracture is hydraulically driven with the injection of a fluid from the wellbore into the fracture channel. In this model, a pre-defined surface made up of elements that support the cohesive zone traction-separation calculation is embedded in the rock and the hydraulic fracture grows along this surface. The fracture process zone (unbroken cohesive zone) is defined within the separating surfaces where the surface tractions are nonzero. The fracture is fully filled with fluid in the broken cohesive zone where no traction from rock fracture exists, but where fluid pressure is acting on the open fracture surfaces. The definition of the crack tip as used in reference (Shet and Chandra, 2002) is adopted here, the mathematical crack tip refers to the point which is yet to separate; the cohesive crack tip corresponds to the damage initiation point where the traction reaches the cohesive strength $T_{\max}$ and the separation reaches the critical value δ_0; the material crack tip is the complete failure point where the separation reaches the critical value δ_f and the traction or cohesive strength acting across the surfaces are equal to zero. The fracturing fluid can permeate the cohesive damage zone. Thus the fluid front is taken to coincide with the cohesive crack tip.

2.1. The cohesive law

The cohesive law defines the relationship between the traction tensor **T** and the displacement jump **δ** across a pair of cohesive surfaces. A cohesive potential function ψ is defined so that the traction is given by

$$\boldsymbol{T} = \frac{\partial \psi}{\partial \boldsymbol{\delta}}. \tag{1}$$

Various types of traction-separation relations (potential functions) for cohesive surfaces have been proposed to simulate the fracture process in different types of material systems. The irreversible bilinear cohesive law (Tomar et al., 2004), as shown in Fig. 2, is adopted in this study. This bilinear law is a special case of the trapezoidal model. It can also be regarded as a generalised version of the initial rigid, linear-decaying irreversible cohesive law. It has been widely used to simulate the fracture or fragmentation processes in brittle materials.

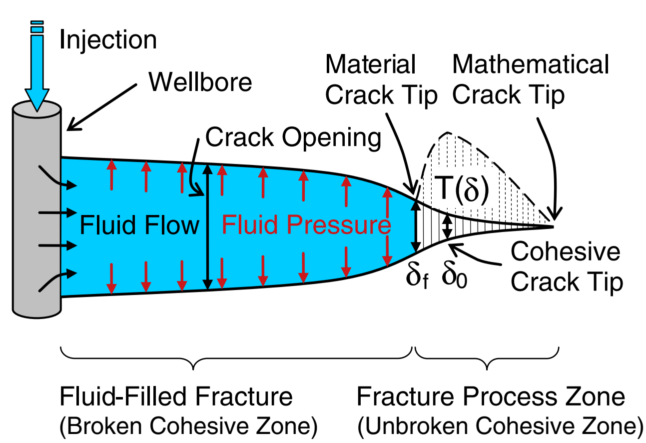

Fig. 1. Embedded cohesive zone in a hydraulic fracture.