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Analytical modeling of linear flow with variable permeability distribution in tight and shale reservoirs

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ABSTRACT

Horizontal wells with multiple fractures, producing from tight and shale reservoirs, may exhibit linear flow for long periods of time. Majority of the available analytical solutions that model this flow behaviour assume uniform permeability, which most likely is an over-simplification of the unconventional reservoirs. Non-homogeneous shear or tensile failure away from the main induced (primary) hydraulic fractures can lead to a non-uniform permeability distribution that depends on the distance away from the hydraulic fractures.

In this work, linear flow in a reservoir with non-uniform permeability adjacent to the primary hydraulic fracture is modeled rigorously using perturbation theory. The diffusivity equation is solved for the pressure response of a fractured well located in a reservoir of infinite extent with permeability as an arbitrary function of position. For constant terminal rate (CTR) or constant terminal pressure (CTP) conditions, the linear flow parameter ($LFP = x_f\sqrt{k}$) is calculated from the slope of the square-root-of-time plot (plot of rate-normalized pressure vs. square root of time).

It is demonstrated herein that the calculated *LFP* corresponds to a weighted average of permeabilities (and fracture half-lengths); different parts of the reservoir contribute differently to the *LFP* at different production times. The *LFP* is influenced most strongly by permeabilities at a distance $y = 0.056\sqrt{\frac{(kt)}{\phi\mu c_t}}$. The derived weighting functions during CTR and CTP production can be applied in inverse mode for determining *LFP* distribution near the hydraulic fractures. This is particularly useful in evaluating the effectiveness of hydraulic fracturing operations and assessing the performance of different fracturing techniques in unconventional reservoirs. In addition, this work gives significant insight into the concept of distance of investigation (DOI) in tight and shale reservoirs, and the differences when producing under CTR and CTP conditions.

1. Introduction

Tight and shale oil and gas reservoirs have recently attracted the industry's attention because of their extent and the recently-acquired technological ability to extract them. Irrespective of the hydrocarbon fluid type (oil and/or gas) and reservoir type (tight sandstone or carbonates, shales or a combination) the permeability of tight/shale oil/gas reservoirs is low to ultra-low, on the order of micro- to nano-darcies in some cases. Thus, a key strategy for their economic production is to create high-permeability pathways from the reservoir to the well through massive hydraulic fracture stimulation treatments. In most cases, depending on a variety of factors including in-situ stress magnitude and orientation, rock fabric etc., an enhanced permeability region around primary hydraulic fractures may be created. Such a region is commonly called Stimulated Reservoir Volume, or SRV (Mayerhofer et al., 2010). The permeability enhancement in the SRV originates from rejuvenation of existing natural fractures and/or development of a complex fracture “network” during stimulation. These natural (and induced) fractures lead to an altered system with a permeability distribution that is not uniform (Fuentes-Cruz et al., 2014). It is the combined (averaged) effect of this non-uniform fracture network over an extensive region around the horizontal wellbore that makes production from unconventional reservoirs viable.

Production data (and welltest) analysis of wells completed in tight and shale reservoirs can exhibit linear flow for long periods of time. Analysis of long-term linear flow therefore can be used as a means of obtaining valuable information about stimulation efficiency. One of the most popular methods for analyzing linear flow is the square-root-of-time plot, i.e. a plot of rate-normalized pressure (or pseudo-pressure) versus square root of time (or pseudo-time). The slope of this line is

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used to calculate the linear flow parameter ($LFP = x_f\sqrt{k}$). In tight and shale reservoirs, the linear flow parameter (*LFP*) fuses the effect of the created complex fracture network (through the x_f term), with an induced permeability field (through the k term). This means that the *LFP* is the result of an averaging process taking place over a region within the reservoir; further it describes the effectiveness of the created fracture network. Available approaches for linear flow analysis assume uniform permeability throughout the reservoir (Clarkson, 2013; Nobakht and Clarkson, 2012a,b; Wattenbarger et al., 1998; El-Banbi and Wattenbarger, 1998). While this is a useful simplifying assumption for a first-pass linear flow analysis, it is of practical significance to consider the non-uniform nature of the induced SRV and its effect on linear flow analysis. This is especially useful for gaining a detailed understanding of the flow performance in tight and shale reservoirs, and the averaging process taking place during production, in addition to obtaining an improved characterization around the created hydraulic fractures.

Previously, Oliver (1990) investigated the averaging process of permeability through well test analysis of a well located in an infinite reservoir (radial flow regime) with variable permeability. The averaging process was quantified through an equation that relates the instantaneous semilog slope to a volume integral of permeability variation multiplied by a weighting function. He derived analytic expressions for the weighting (and Kernel) functions as a function of time and distance, and noted that the welltest permeability estimate is influenced most strongly by permeabilities at a distance $r = 0.015(kt/(\phi\mu c_t))$. The Kernel function has been used as the basis for performing welltest analysis in radially heterogeneous reservoirs (Feitosa, 1993; Feitosa et al., 1994; Sangsoo, 1995; Sagar et al., 1995). In addition, it has been used in many numerical well studies to provide the well test response of various permeability realizations without running the flow simulation (Gautier and Noetinger, 2004; Hamdi et al., 2013; Hamdi, 2014). Analogous to Oliver's work, in this paper the perturbation method is used to solve for the pressure response of an infinite reservoir with variable permeability. Perturbation theory can give an approximate description of a complex system by employing some perturbations from the response of a solvable equivalent ideal system (Wiesel, 2010). In addition, a study of the averaging process for the CTR and CTP linear flow regime, including identification of the region of the reservoir that influences the *LFP* estimates, and a specification of the relative contribution of the permeability of various regions to the estimate of average *LFP*, is performed. *LFP* estimates are shown to be the result of a weighted harmonic averaging of the permeabilities and fracture half-lengths of some reservoir volumes, similar to the radial flow permeability estimates from the slope of a semilog plot (Oliver, 1990). Comparison of the Weighting/Kernel functions (and therefore the averaging process) between the CTR and CTP linear flow sheds light into the DOI equations obtained for these two production scenarios. Despite a number of research conducted on the topic (El-Banbi and Wattenbarger, 1998; Nobakht and Clarkson, 2012a,b; Behmanesh et al., 2015), the differences in DOI for these two end-member operating conditions during transient linear flow have never been adequately explained until now.

2. Mathematical development

The basic reservoir model and the element of symmetry considered in this work is illustrated in Fig. 1. Fig. 1a shows the fractured horizontal well stimulated in multiple fracturing stages, and Fig. 1b shows the permeability enhancement around the hydraulic fracture. Based on the physical consideration, it is reasonable to use the element of symmetry as shown in Fig. 1c. This is similar to the fractured well/reservoir configuration used by Wattenbarger et al. (1998), Nobakht and Clarkson (2012a,b) and Shahamat et al. (2015), which exhibits transient linear flow at early time. The difference here, however, is that the reservoir permeability is assumed to vary with distance from the hydraulic fracture according to a permeability distribution, $k_D(y_D)$. This permeability distribution is meant to represent the situation where permeability is elevated in the near fracture/wellbore region by varying degrees, depending on the distance to the hydraulic fracture. Hummel and Shapiro (2013) used microseismic data from the Barnett shale reservoir to demonstrate that the generation of hydraulic fractures leads to a nonlinear permeability distribution in the SRV with the largest permeability magnitudes occurring in regions closer to the wellbore.

The main purpose of this paper is to find the analytical solutions for the pressure response of the fractured well model shown in Fig. 1 using the non-uniform permeability distribution $k_D(y_D)$. Development of the analytical solutions is based on the assumption of single-phase flow of a slightly compressible fluid with negligible wellbore storage and skin effects.

The dimensionless diffusion equation for the linear flow (Shahamat et al., 2015) can be modified to account for variable permeability distribution. This is achieved through introduction of the dimensionless permeability term $k_D(y_D)$ in the following form:

$$\frac{\partial}{\partial y_D}\left(k_D(y_D)\frac{\partial p_D}{\partial y_D}\right) = \frac{\partial p_D}{\partial t_D} \tag{1}$$

In Eq. (1), the permeability distribution function can be written as the sum of a constant "average" permeability and a variation of the constant value. In this analysis, we assume that the permeability variation is much smaller in magnitude than the average permeability, that is:

$$k_D(y_D) = \frac{1}{1 - \varepsilon f(y_D)} \tag{2}$$

where $f(y_D)$ is the permeability-variation function and ε is a small real number. It is demonstrated in the Applications Section that Eq. (2) provides a simple form for the permeability distribution that is mathematically convenient and meaningful.

3. Solutions

If the permeability variation from the average value is small, perturbation theory can be applied to approximate the solutions for the pressure at the wellbore by the sum of an infinite series of terms of rapidly decreasing magnitude. Retaining the first two terms in the series gives:

$$p_D = p_{D0} + \varepsilon p_{D1} + O(\varepsilon^2) \tag{3}$$

Here the p_{D0} is the pressure response for a constant (average) permeability reservoir that can be approximated by the square root of time (Wattenbarger et al., 1998). Moreover, p_{D1} is the first perturbation to the constant-permeability solution and is a function of the permeability variation function, $f(y_D)$.

In the following subsections, the analytical solutions of Eq. (1) for CTR and CTP are obtained using Laplace transforms with subsequent inversions to the time space, and a perturbation expansion in powers of ε. Details of the derivations are given in Appendices A and B.

3.1. Constant terminal rate (CTR) production

The dimensionless parameters used for obtaining Eq. (1) in CTR production, along with detailed derivation of the solution for the dimensionless pressure in Laplace domain, are given in Appendix A. Inversion of the solution into the real time domain results in the following equation:

$$p_{wD} = \sqrt{\pi t_D} - \varepsilon\int_0^\infty f(y_D)G(y_D, t_D)dy_D \tag{4}$$

where $G(y_D, t_D)$ is the weighting function for the permeability distribution and is equal to: