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Numerical modeling of unsteady-state wellbore heat transmission

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ABSTRACT

This paper proposes a fully-implicit thermal model, which couples the wellbore and the surrounding formation. The authors use this integrated model to simulate and predict the unsteady-state heat transmission and temperature distribution in the wellbore and the surrounding formation under variable operational constraints. This model employs refined grid discretization for both the wellbore and the surrounding formation domains. Detailed mass, energy and momentum balance analysis is given for each grid block, which is solved simultaneously at every time step. The model captures the near-wellbore boundary effects using geometric spacing. The resulting set of equations is solved by the Unsymmetric MultiFrontal solver (UMFPACK). We compared and validated the results of the numerical model, against both a conventional Ramey's approach and a rigorous analytical solution introduced by Hagoort. The results of the comparison show that this model is able to predict the temperature distribution in the wellbore and the surrounding formation from early to late time. Contrasted with Ramey's approach, this model yields more accurate results at any time scale, particularly for early time values when the wellbore temperature profile is determined by the wellbore inlet fluid frontier. Two case studies are discussed to test the feasibility of this model under variable operating conditions. The results of this procedure indicate that this model is applicable to both variable-temperature and multi-rate injection cases. The temperature profiles in the wellbore and the surrounding formation are presented with temperature contours, displaying how the temperature changes from near the wellbore to deep in the formation. Moreover, a superposition time is defined and integrated with Ramey's approach (as an alternative to the simulation model) in order to produce a quick and relatively accurate prediction of the wellbore temperature profile. This approach has wide application in cyclic steam-injection and geothermal wells under variable operating conditions.

1. Introduction

Studies on well heat transmission have appeared repeatedly in the literature. Reliable modeling for injection or production wells is essential for estimation of temperature change both in the wellbore and in the surrounding formation. The majority of these studies go back to Ramey (1962) where he presented a model based on a simplified heat balance to simulate heat transmission during the injection of hot fluids. He assumed that heat transmission in the wellbore is steady state, neglecting an accumulation term in the wellbore and proposing a simplified approach to estimating the transient heat transmission in the surrounding formation. His model works well over long time scales. Nevertheless, at early stages, the simplification would yield to considerable errors.

Not long after this pioneering work, Satter (1965) improved Ramey's model by considering the condensation of the injected fluid, which extended Ramey's model to steam-injection processes. He replaced the overall heat coefficient with a depth-dependent factor. Willhite (1967) then published a complete method to estimate the overall coefficient of heat transfer from wellbore to formation, by introducing terms based on different component materials such as casing, insulation and cement. A simplified procedure was also introduced to calculate the overall heat transfer coefficient.

Subsequently, different models were developed, based on previous work, to predict the heat loss and temperature profile in various cases, especially in cases of hot fluid injection (steam or hot water). Farouq Ali (1981) proposed a mathematical model to simulate the upward or downward flow in a geothermal wellbore, while employing Ramey's method to calculate the heat loss from

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wellbore to formation. One year later, Fontanilla and Aziz (1982) modeled the performance of steam injection. Their model considered a two-phase flow in the wellbore and used Willhite's overall heat coefficient when calculating heat loss from the wellbore to formation, assuming that heat transmission in the wellbore reaches a steady state in a very short period of time. Stone et al. (1989) proposed a fully implicit model to simulate the fluid flow inside both the wellbore and the formation by formulating mass, energy and momentum equations. Wu and Pruess (1990) introduced an analytical model to simulate wellbore heat transmission with a uniformly layered surrounding formation. Their results showed that Ramey's method would generate considerable errors in early time periods. Sagar et al. (1991) presented a simple model to predict temperature changes in two-phase flowing wells; they compared the results of their model with field data to validate its applicability in a wide range of conditions. However, their model was formulated based on the assumption that the wellbore is under steady state conditions. Bahonar et al. (2010) developed a numerical model to predict sandface conditions in wet steam-injection processes. Their model formulated mass and momentum equations inside the wellbore, employing a drift flux model to address the two-phase flow pressure drops. Nonetheless, because the wellbore accumulation term is neglected, the whole model is solved under semisteady-state conditions. Most of the aforementioned models employed Ramey's approach to estimate heat loss by using an overall transient heat coefficient and solving heat transmission in the wellbore as a steady state process. This may lead to some errors in early times, while relatively good results over long time scales.

Hagoort (2004) developed a rigorous solution for heat transmission in the wellbore, without simplifications, and compared his solution with Ramey's approach. He demonstrated similar results at long time scales, but Hagoort's model yielded more accurate early-time results than Ramey's. Due to its reasonable description of the transient wellbore heat transmission, Hagoort's model was chosen to compare and validate results of the new numerical model. Although Hagoort's model performs well in transient heat transfer prediction, it is limited to fixed well constraints (e.g. a fixed injection rate or a fixed temperature). Hagoort's model does not allow for variable injection temperatures or production rates. In practice, however, the prediction of the temperature profiles under continually varying conditions is a recurring problem (e.g. cyclic steam stimulation).

In the following approach, a numerical formulation is provided in order to simulate correctly and predict transient wellbore heat transmission under continuously changing operating conditions (e.g. injection temperature and flow rates).

2. Model formulation

The physical model is divided into three sections: (i) tubing, (ii) annulus and casing, (iii) surrounding formation. Heat flow within the tubing is considered unsteady state, single phase, 1-D in the vertical direction and incompressible. Energy and momentum balance equations have been coupled in cylindrical coordinates in order to describe the system mathematically.

Heat conduction through the formation surrounding the wellbore can be expressed by the radial Fourier heat conduction equation:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T_e}{\partial r}\right)+\frac{\partial}{\partial z}\frac{\partial T_e}{\partial z}=\frac{1}{\alpha}\frac{\partial T_e}{\partial t} \tag{1}$$

$$\alpha=\frac{k}{C_p\rho_{earth}} \tag{2}$$

where T_e is formation temperature, k is formation heat conductivity, C_p is specific heat of the rock formation and ρ_{earth} is rock formation density. The initial and boundary conditions are the following:

$$t=0: r\geq r_{cf}: \quad T_e=T_{surface}+g_T z \tag{3-a}$$

$$t>0: r=r_{cf}: \dot{Q}=2\pi r_{cf}U(T_b-T_e)=\frac{1}{R_h'}(T_b-T_e) \tag{3-b}$$

$$t>0: r=\infty: T_e=T_{surface}+g_T z \tag{3-c}$$

where $T_{surface}$ is initial temperature of the formation's surface, g_T is the earth's thermal gradient, r_{cf} is the outer radius of the wellbore, and U is the near-wellbore heat transfer coefficient.

Neglecting heat loss caused by friction, the tubing heat balance (Ramey, 1962; Ali, 1981) is the following:

$$\frac{\partial}{\partial t}\left[A\rho_f\left(H+\frac{v^2}{2g_cJ_c}-\frac{g}{g_cJ_c}z\right)\right]+\frac{\partial}{\partial z}\left[i_v\rho_f\left(H+\frac{v^2}{2g_cJ_c}-\frac{g}{g_cJ_c}z\right)\right]=-\dot{Q} \tag{4}$$

where A is the cross section area of the tubing, ρ_f is the fluid density, H is the specific enthalpy, v is the liquid flow velocity, and i_v is the flow rate.

For incompressible fluid, the specific enthalpy is:

$$dH=C_pdT+\frac{d(PV)}{J_c} \tag{5}$$

By neglecting friction, the $d(PV)$ term will equal the change of fluid hydrostatic pressure, thus:

$$dH=C_pdT+\frac{g}{g_cJ_c}dz \tag{6}$$

Taking Eq. (6) into Eq. (4) and neglecting the kinetic energy term, the resulting equation can be simplified as:

$$A\rho_fC_{pf}\frac{\partial T}{\partial t}+\rho_fC_{pf}i_v\frac{\partial T}{\partial z}=-\dot{Q} \tag{7}$$

Initially, the temperature in the tubing is equal to that of the surrounding formation, which can be expressed as:

$$t=0: 0\leq z\leq L: T_b=T_{surface}+g_T z \tag{8}$$

For an injection well, the temperature at the wellhead is the injection temperature (T_{inj}):

$$t>0: z=0: T_b=T_{inj} \tag{9-a}$$

For a production well, the temperature at the entrance of the tubing is equal to the reservoir temperature (T_{prod}):

$$t>0: z=L: T_b=T_{prod} \tag{9-b}$$

The momentum balance equations (Ali, 1981; Hasan et al., 2007, 2003; Livescu et al., 2008, 2010; Mcmillan, 2011; Beggs and Brill, 1973) will be as follows.

For an injection well: