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Mixing of ultrasonic Lamb waves in thin plates with quadratic nonlinearity

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ABSTRACT

This paper investigates the propagation of Lamb waves in thin plates with quadratic nonlinearity by one-way mixing method using numerical simulations. It is shown that an A_0-mode wave can be generated by a pair of S_0 and A_0 mode waves only when mixing condition is satisfied, and mixing wave signals are capable of locating the damage zone. Additionally, it is manifested that the acoustic nonlinear parameter increases linearly with quadratic nonlinearity but monotonously with the size of mixing zone. Furthermore, because of frequency deviation, the waveform of the mixing wave changes significantly from a regular diamond shape to toneburst trains.

1. Introduction

Due to the accuracy and sensitivity limitations of the conventional nondestructive testing techniques [1–3], these methods cannot be employed to quantitatively detect and evaluate early material degradation. However, with the development of electronics and theoretical nonlinear acoustics, nonlinear ultrasonic techniques have attracted a growing amount of attention [4–10]. Many experiments [11–13] have revealed that the nonlinear elastic behavior of the material constitutive relationship arises from the microstructural changes or discontinuities, such as micro-cracks [14–16], dislocations [17] and precipitates [18]. Based on different nonlinear effects, a number of nonlinear ultrasonic techniques have been developed, including higher harmonics technique [19], sub-harmonic and DC response technique [20], nonlinear resonant ultrasound spectroscopy technique, and mixing resonant wave technique [21–23].

Because of the long propagation distance of Lamb waves in a thin plate structure, nonlinear Lamb-wave detection techniques have attracted more and more attention in recent years [11,19,24–28]. Many efforts have been devoted to characterizing the early material degradation by higher harmonics of Lamb waves. Deng et al. [24,25] have reported that obvious second harmonic can be observed only when the phase velocities of the fundamental and double frequency Lamb waves match with each other. In most studies, the S_1 and A_1 [10,27] modes are selected as the fundamental waves as the phase velocity matching can be easily matched for these two modes. Definitely, the S_0 and A_0 modes [16,28,29] have the great advantages because of higher energy. Hu et al. [16] have performed numerical simulations on the propagation of the low frequency S_0 Lamb wave in thin plates with randomly distributed micro-cracks to analyze the behavior of nonlinear Lamb waves.

Traditional nonlinear wave techniques such as higher harmonics technique cannot locate the position of the early material degradation. Recently, nonlinear wave mixing techniques with non-collinear [30,31] or collinear [21,23,32,33] methods, have been applied to detect the change of material nonlinearity. This technique has some unique advantages compared with traditional nonlinear wave techniques. For example, by controlling wave-mixing locations, researchers can scan over the regions of interest; frequencies can be selected to avoid unwanted harmonics caused by some electronic components in a measurement system; and local nonlinearities in the mixing zone can be represented instead of the average value during the incident wave propagation. For bulk waves, the collinear wave mixing technique has been studied analytically and experimentally. For instance, Chen et al. [21] have derived a set of necessary and sufficient conditions for generating

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resonant waves by two propagating time-harmonic plane waves and obtained closed-form analytical solutions for resonant waves generated by two collinearly propagating sinusoidal pulses. Zhao et al. [23] have investigated the frequency deviation during imperfect resonant conditions through FEM simulations and experimental measurements using a one-way mixing technique. Liu et al. [34] have developed a collinear wave mixing method to measure the acoustic nonlinearity parameter. Tang et al. [33,35] have developed a collinear mixing based scanning method to detect the localized plastic deformation.

However, studies on the mixing behavior of Lamb waves have been rarely reported. Therefore, we aim to investigate and evaluate the mechanism of Lamb wave mixing in thin plates with quadratic nonlinearity by one-way mixing method in this paper. The condition for generating Lamb mixing waves is obtained by numerical modelling of a pair of S_0 and A_0 mode waves, and the quantitative relationship between the acoustic nonlinear parameter and the characteristics of quadratic nonlinearity are also studied. This work may provide a feasible theoretical basis for nonlinear Lamb-wave detection techniques based on one-way mixing method.

2. Mixing condition for Lamb waves

Lamb waves are most commonly used in the acoustic guided-wave modes for thin plate structures, whose wavelength is of the same magnitude as the thickness of the plate. The dispersion is the most significant characteristic of the Lamb waves, that is, the velocity of propagation depends on not only the elastic constants and density of the material, but also the frequency of the waves. Fig. 1 shows the dispersion curves of the Lamb wave propagation in a 2 mm thick aluminum plate.

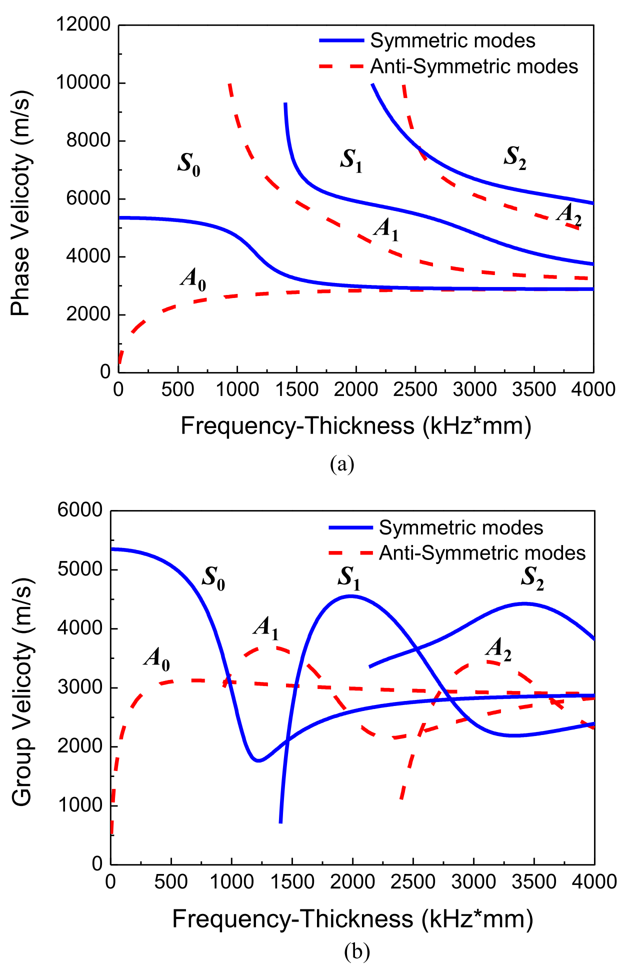

Fig. 1. Dispersion curves of Lamb waves in a 2 mm thick aluminum plate: (a) phase velocity; (b) group velocity.

Chen et al. [21] derived the necessary and sufficient resonance conditions for bulk waves. When a longitudinal wave pulse and a transverse wave pulse are both emitted at $x = 0$ and propagate in the positive x-direction, it is called one-way mixing. If the resonance condition $\omega_L/\omega_T = 2\kappa/(\kappa+1)$ can be satisfied, a resonant transverse wave will be generated which propagates in the opposite direction of the primary transverse wave. The circular frequency of the resonant waves is $\omega_R = \omega_L - \omega_T$, where, $\kappa = c_L/c_T$; c_L and c_T are the longitudinal and transverse phase velocities, respectively; ω_T and ω_L are the circular frequencies of the longitudinal and transverse waves, respectively.

Thus, we propose that, if the mixing condition $\omega_{S_0}/\omega_{A_0} = 2\kappa/(\kappa+1)$ for one-way Lamb wave mixing is satisfied, an A_0 mode Lamb wave can be generated and propagates in the opposite direction of the incident A_0 mode Lamb wave. The circular frequency of the mixing wave is $\omega_R = \omega_{S_0} - \omega_{A_0}$, where, $\kappa = c_{S_0}/c_{A_0}$; c_{S_0} and c_{A_0} are the phase velocities of the S_0 and A_0 mode Lamb waves, respectively; ω_{S_0} and ω_{A_0} are the circular frequencies of the S_0 and A_0 mode Lamb waves, respectively.

3. Numerical simulation

Numerical simulations are employed to validate the above assumption. A two-dimensional finite element model (FEM) of a thin plate structure with quadratic nonlinearity is built using the commercial FEM software ABAQUS (Version 6.14, Dassault Systèmes Simulia Corp., Providence, RI, USA).

By employing a pair of S_0 and A_0 mode Lamb waves, the problem of one-way mixing of Lamb waves in thin plates with quadratic nonlinearity is described in Fig. 2. To illustrate how to locate the damaged region, a different schematic will be shown in Section 4.3. An S_0 mode wave pulse and an A_0 mode wave pulse are generated using a dynamic displacement excitation on the left end of the model. Because of the different phase velocities between these two pulses, the A_0 mode wave pulse needs to be generated ahead of the S_0 mode wave pulse. These two pulses propagate along the positive direction of x, and interact mutually in the mixing zone. If the mixing condition is satisfied, the A_0 mode wave can be generated and propagates along the negative direction of x, which is finally received at the detection positions.

The quadratic nonlinear elastic constitutive law including second-order constants and third-order constants is used [23], which is expressed using Voigt's notation $C_{ijkl} = c_{IJ}$, $C_{ijklmn} = c_{IJK}$:

$$\sigma_{ij} = C_{ijkl}E_{kl} + \frac{1}{2}C_{ijklmn}E_{kl}E_{mn} \tag{1}$$

$$C_{ijkl} = \lambda\delta_{ij}\delta_{kl} + \mu(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}) \tag{2}$$

$$C_{ijklmn} = (2l - 2m + n)\delta_{ij}\delta_{kl}\delta_{mn} + (2m - n)(\delta_{ij}I_{klmn} + \delta_{kl}I_{mnij} + \delta_{mn}I_{ijkl}) + \frac{n}{2}(\delta_{ik}I_{jlmn} + \delta_{il}I_{jkmn} + \delta_{jk}I_{ilmn} + \delta_{jl}I_{ikmn}) \tag{3}$$

where l, m and n are the Murnaghan third-order elastic constants, E is the Lagrangian or Green strain, $I, J, K \in \{1,2,3,4,5,6\}$, $ij = 11,22,33,23,31,12 \leftrightarrow I = 1,2,3,4,5,6$.

The material properties of the aluminum plate (AL-6061-T6) [28] used in the simulations are listed in Table 1. Since at least 20 elements are required in the wavelength of the highest frequency waves for calculation precision, the appropriate element size is set to L_{max} = 0.2 mm. Thus, 100,000 four-node plane strain