Ultrasonics 84 (2018) 244–253

Contents lists available at ScienceDirect

Ultrasonics

journal homepage: www.elsevier.com/locate/ultras

Shear horizontal wave propagation in a periodic stubbed plate and its application in rainbow trapping

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ARTICLE INFO

Article history:
Received 14 October 2017
Received in revised form 15 November 2017
Accepted 15 November 2017
Available online 16 November 2017

Keywords:
Shear horizontal waves
The high-order waveguide mode theory
Band structures
Rainbow trapping

ABSTRACT

The high-order waveguide modal theory, usually used in electromagnetics and acoustics, is adopted to investigate the propagation properties of shear horizontal waves in a periodic stubbed plate. Beyond the sub-wavelength regime, higher-order modes are included to calculate the exact band structures caused by the stubs. Theoretical solutions are obtained in a closed form, in which both the dynamic governing equations and the boundary conditions are strictly satisfied. It is shown that the proposed modelling approach exhibits good convergence and accuracy, in agreement with results obtained from the finite element method. After a systematic investigation on the influence of the stub on the evolution of the band structures, the so-called rainbow trapping phenomenon of SH waves is revealed and explored in a graded stubbed plate with monotonously increasing height or width of the stubs, featuring an obvious reduction of the group velocity and blocking of the wave propagation at different locations for SH waves of different frequencies. The proposed model is expected to provide a useful theoretical tool for the physical mechanism exploration, structural design and eventually system optimization to guide various engineering applications of SH waves.

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1. Introduction

As artificially structured composite materials, acoustic metamaterials (AMs) and phononic crystals (PCs) exhibit anomalous physical properties that cannot be found in nature. Typical examples include absolute band gaps (BGs) [1–3], directional BGs for unidirectional transmissions [4], negative refractions for wave focusing [5], zero-angle refraction for wave collimations [6] and so forth. The diverse functionalities of the AMs and PCs are being explored for various applications, such as cloaking [7], phase manipulation [8], sound absorption [9] and active control [10,11]. The ultimate aim is to be able to manipulate wave propagations through structural design. Conventional PCs and AMs usually consist of two-phase or multi-phase components to create an impedance mismatch as a result of the differences in material properties. Alternatively, the impedance mismatch can also be generated through varying the structural shape or other geometric parameters [12–14]. Acoustic black hole (ABH) structures with the structural thickness tailored in a particular form is a typical example, in which bending waves can be controlled artificially [15,16]. Another example is the periodically corrugated structures made of the same material in one piece [17], which provides a simple and potential substitution for wave devices, since less design parameters are involved. The advent of new manufacturing capabilities such as 3D printing also makes it possible to fabricate structural components with more complex geometries. Such designs also avoid the joints between multi-phase materials, which are not desirable in manufacturing, assembly and applications.

For the control and the manipulation of elastic waves in a geometry-induced inhomogeneous medium, the primary task is to be able to accurately predict the band structures expressed in terms of the frequency spectrums. Mathematically, it can be expressed in a general form as $G(f) = 0$, in which G stands for an explicit or implicit function of the frequency f. From the mechanical viewpoint, the generalized form of $G(f)$ can be deduced by satisfying the dynamic equations and the corresponding Bloch theorem as the necessary boundary conditions between the unit cells in a periodic structure. However, mathematically, the full dynamic equations describing the elastic wave propagation in solids are governed by the displacement vector, in which the inherent mode coupling needs to be considered. This creates a tremendous challenge for the establishment of the theoretical model allowing for analytical solutions. Hence, most investigations on elastic waves in AMs and PCs have been based on numerical simulations and experiments [12,13,18]. Theoretically, the band structures caused by geometry-induced

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https://doi.org/10.1016/j.ultras.2017.11.010
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mismatched impedance can be calculated via the revised plane-wave-expansion method [16,19,20], in which virtual vacuum layers need to be added in order to satisfy the traction-free surfaces over the structural portion where geometrical changes take place. In so doing, solutions depend on the thickness of the vacuum layers, and the convergence of the solution is sometimes rather poor. Besides, homogenization methods [21,22] can also be used to obtain the band structure through calculating the effective constitutive parameters of the complex materials, which is known as an efficient tool only for long wavelength approximation. Therefore, theoretical or semi-analytical models, capable of accurately describing the elastic wave propagation in AMs and PCs, are highly desirable. This motivates the present contribution.

Inspired by works in electromagnetics and acoustics [23–26], the high-order waveguide modal theory is utilized to establish a theoretical model on the shear horizontal (SH) wave propagation in a periodic stubbed plate. The model allows for an analytical solution, in which the Bloch theorem is included automatically, which provides much convenience for the mathematical derivation. In the proposed model, higher-order modes are included to get the exact band structures caused by the stubs. The proposed model and the solution show fast convergence by using a small number of terms and high accuracy through comparisons with the result from the finite element method (FEM). Numerical analyses reveal the so-called rainbow trapping phenomenon of SH wave in a graded stubbed plate with monotonously increasing height or width of the stubs, featuring an obvious reduction of the group velocity and blocking of wave propagation at different locations for SH waves of different frequencies.

2. The high-order waveguide mode theory

Consider a periodic stubbed plate with two different additional partial stubs on its upper and bottom surfaces. The plate is assumed unbounded in the x_3 direction, and only a cross section of the unit cell from the stubbed plate is shown in Fig. 1. For convenience, the inhomogeneous plate is divided into three homogeneous regions. Region I is the middle flat plate between the two stubs, whose thickness and periodicity are denoted by $2h$ and l, respectively. Regions II and III correspond to the upper and bottom stubs, occupying the region $|x_2| \leqslant 0.5w^{\mathrm{up}}$ and $|x_2| \leqslant 0.5w^{\mathrm{down}}$ and having a height of d^{up} and d^{down}, respectively. When a SH wave travels in the inhomogeneous plate, its propagation properties are altered by the geometry-induced mismatched impedance, which will be the focus of the analyses.

The dynamic equation governing the SH waves only involves a displacement component in x_3 direction $u_3 = u(x_1, x_2)$ as

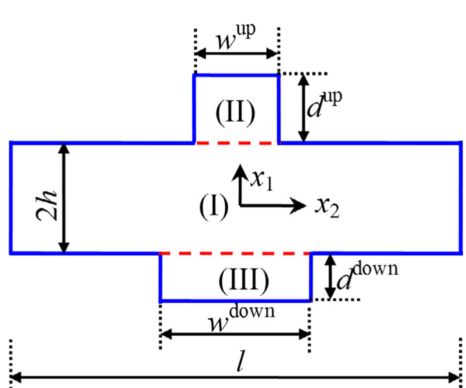

Fig. 1. Scheme of a unit cell from a periodic stubbed plate.

$$\mu\left(\frac{\partial^2 u}{\partial x_1^2}+\frac{\partial^2 u}{\partial x_2^2}\right)=\rho\frac{\partial^2 u}{\partial t^2}, \tag{1}$$

in which μ and ρ are, respectively, the shear modulus and mass density, and t is the time. Given an incident harmonic SH plane wave with an angular frequency ω in the plate, the wave field in the middle plate denoted by Region I can be expressed as [23–27]

$$u^{(\mathrm{I})}=\sum_{\gamma=-\infty}^{\infty}\left[A_\gamma\cos\left(k_{1,\gamma}x_1\right)+B_\gamma\sin\left(k_{1,\gamma}x_1\right)\right]\exp\left(ik_{2,\gamma}x_2\right), \tag{2}$$

in which A_γ and B_γ are the coefficients to be determined and the common term, $\exp(i\omega t)$, has been omitted for brevity. Here, the incident wave field is dropped in Eq. (2), since the dispersion relation, instead of forced vibration, is of the primary concern. As a matter of fact, Eq. (2) can be regarded as the diffraction field caused by the stubs, which is analogue to electromagnetics and acoustics. The eventual inclusion of the incident wave field in Eq. (2) allows the calculation of the reflection and transition coefficients in photonics and phononics [23–26], in which 0-order waveguide mode is usually applied for the sub-wavelength regime. However, beyond the sub-wavelength regime, sufficient higher-order modes should be included in Eq. (2) in order to exactly describe the wave propagation properties of the SH waves. $k_{1,\gamma}=\sqrt{\frac{\omega^2}{c_{\mathrm{SH0}}^2}-k_{2,\gamma}^2}$ stands for the wave number in x_1 direction with the bulk velocity $c_{\mathrm{SH0}}=\sqrt{\frac{\mu}{\rho}}$. Meanwhile, considering the Bloch theory, the γ-order diffraction wave vector in x_2 direction can be denoted by $k_{2,\gamma}=k+\frac{\gamma 2\pi}{l}$ with k ranging from $-\pi/l$ to π/l in the irreducible Brillouin zone [27,28]. Based on the solution, the shear stress component can be obtained as

$$T_{31}^{(\mathrm{I})}=\sum_{\gamma=-\infty}^{\infty}\mu k_{1,\gamma}\left[-A_\gamma\sin\left(k_{1,\gamma}x_1\right)+B_\gamma\cos\left(k_{1,\gamma}x_1\right)\right]\exp\left(ik_{2,\gamma}x_2\right). \tag{3}$$

The displacement of the SH wave can be expressed by virtue of trigonometric function expansion technique [27,29]. For the upper and bottom stubs, the displacement fields can be written as

$$u^{(\mathrm{II})}=\sum_{n=0,1,2}^{\infty}C_n\left[e^{iq_n^{\mathrm{up}}\left(h+d^{\mathrm{up}}-x_1\right)}+e^{-iq_n^{\mathrm{up}}\left(h+d^{\mathrm{up}}-x_1\right)}\right]\cos\left[\alpha_n^{\mathrm{up}}\left(x_2+0.5w^{\mathrm{up}}\right)\right] \tag{4}$$

$$u^{(\mathrm{III})}=\sum_{m=0,1,2}^{\infty}D_m\left[e^{iq_m^{\mathrm{down}}\left(h+d^{\mathrm{down}}+x_1\right)}+e^{-iq_m^{\mathrm{down}}\left(h+d^{\mathrm{down}}+x_1\right)}\right]\times\cos\left[\alpha_m^{\mathrm{down}}\left(x_2+0.5w^{\mathrm{down}}\right)\right], \tag{5}$$

where C_n and D_m are the coefficients to be determined. $\alpha_n^{\mathrm{up}}=\frac{n\pi}{w^{\mathrm{up}}}$ and $\alpha_m^{\mathrm{down}}=\frac{m\pi}{w^{\mathrm{down}}}$ are the wave numbers in x_2 direction for the upper and bottom stubs, respectively. Different m and n stand for different modes, symmetrical when $n=m=0,2,4,\ldots$ and anti-symmetrical when $n=m=1,3,5,\ldots$. Actually, the expressions of $u^{(\mathrm{II})}$ and $u^{(\mathrm{III})}$ embrace the principle of the modal superposition method. It should be noted that the traction free boundary conditions $T_{32}^{(\mathrm{II})}=0$ at $x_2=\pm 0.5w^{\mathrm{up}}$ and $T_{32}^{(\mathrm{III})}=0$ at $x_2=\pm 0.5w^{\mathrm{down}}$ are automatically satisfied in this case. The wave numbers in the upper and the bottom stubs in x_1 direction can be obtained as $q_n^{\mathrm{up}}=\sqrt{\frac{\omega^2}{c_{SH0}^2}-\left(\alpha_n^{\mathrm{up}}\right)^2}$ and $q_m^{\mathrm{down}}=\sqrt{\frac{\omega^2}{c_{SH0}^2}-\left(\alpha_m^{\mathrm{down}}\right)^2}$ by ensuring that Eqs. (4) and (5) satisfy the dynamic governing Eq. (1). Correspondingly, the stress components can be obtained as

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