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A semi-analytical approach for SH guided wave mode conversion from evanescent into propagating

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ABSTRACT

Conversion of evanescent shear horizontal (SH) guided waves into propagating is presented in this paper. The conversion is exemplified by a time-harmonic *SH* evanescent displacement prescribed on a narrow aperture at an edge of a semi-infinite isotropic plate. The conversion efficiency in terms of the amplitude of the propagating *SH* mode converted from evanescent can be expressed in a very simple compact form. The magnitude of the conversion efficiency can be quantified through a derived semi-analytical form based on the complex reciprocity theorem in conjunction with a two-dimensional (2-D) finite element analysis (FEA). Through power conversion analysis, it can be shown that the power flow generated into the plate due to evanescent incident is complex valued. It is theoretically proved that the real part of the complex power flow is associated with the propagating SH modes, while the imaginary part is confined due to the evanescent modes at the plate edge. The conversion efficiency and converted modes are dependent on the geometric configuration of the aperture as well as the selection of the excitation frequency.

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1. Introduction

Wave propagation in isotropic plates can be classified as two types of wave motions: plane strain and antiplane shear motions, where the plane strain motions corresponding to the well-known Lamb waves which are coupled longitudinal and shear vertical waves and the antiplane shear represents shear horizontal (SH) guided waves. *SH* waves are the simplest guided waves in plates, propagating normal to the plane of wave propagation with particle motion in the horizontal plane. A complete dispersion curves including pure imaginary and real wavenumbers of *SH* guided waves in plates can be easily determined analytically [1]. The *SH* waves with pure imaginary wavenumbers which exhibit exponentially decay and thus are called non-propagating or evanescent *SH* waves, while propagating *SH* waves have real wavenumbers.

Because of the unique capabilities of long-range and through-the-thickness interrogation of structures, guided wave based damage detection techniques have been widely used in plate-like structures [2–6], for non-destructive inspection and structural health monitoring. These guided wave modes which interact with defects or different geometric features may generate various wave modes because of mode conversion [7]. To understand the conversion process and quantitatively describe the conversion is essential for reliable damage detection. In addition, it has been known that the reflection of propagating guided waves from the free edge of a plate is accompanied by mode conversion of both propagating and evanescent guided waves, arisen in order to satisfy the traction-free boundary conditions. These localized non-propagating evanescent modes can solely exist in the close proximity of the plate edge, discovered by Torvik [8]. After his pioneer work, many works [9–14] were dedicated to study the propagating Lamb waves reflection/scattering from the free end or structural discontinuities in the plate. The interaction of propagating *SH* guided waves with various structural discontinuities in plates, e.g., free boundary, notch and crack, has been studied by many researchers [15–19]. Recently, Shen and Victor [20] studied the mode conversion of propagating Lamb waves and SH waves extensively. In the literature, these evanescent *SH* guided waves were only considered when studying the mode conversion between propagating guided waves using normal mode superposition methods [21,22] in order to obtain an accurate wavefield distribution around structural discontinuities. Although evanescent *SH* guided waves decay very rapidly, they prevail locally in the near field and may still be converted into propagating waves as they interact with the free boundaries of defects. Therefore, it is of great interest to investigate the conversion of evanescent *SH* guided waves into

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propagating or *vice versa*. Retrieving the information concealed in these evanescent waves from converted propagating *SH* guided waves may provide additional information with respect to the geometric discontinuities or damage. The conversion of evanescent Lamb waves into propagating in plates has been recently demonstrated by Yan and Yuan [23] where the conversion of evanescent into propagating waves was quantitatively investigated.

In this study, a semi-analytical method is adopted to investigate the mode conversion as evanescent *SH* wave is incident through narrow apertures at an edge of a semi-infinite isotropic plate. Characteristics of *SH* guided waves are presented on the basis of the complex dispersion curves. Since *SH* wave exhibits a much simpler analytical solution comparing with Lamb wave, the power flow for propagating and evanescent *SH* mode is thereby analytically determined. Conversion criterion is then discussed to understand the conversion process and the conversion behavior is numerically modeled by solving a partial differential equation directly with the aid of a two-dimensional finite element analysis (FEA). In addition, the amplitude of the converted propagating *SH* mode is obtained in a very concise form by employing the complex reciprocity theorem in conjunction with the FEA. Finally, the conversion efficiency of converting evanescent into propagating *SH* guided waves are quantitatively determined to unveil the underline physics of the conversion.

2. Propagating and evanescent SH guided waves in isotropic plates

2.1. Dispersion relations

In an isotropic plate with surfaces normal to *z* direction, the *SH* wave polarized along *y* direction with displacement *u* alone and the propagation direction is here defined as *x*. The normalized dispersion relation for *SH* guided wave can be simply derived [1] and an analytical form is expressed in terms of non-dimensional variables as

$$\bar{k}^2 = \bar{\omega}^2 - m^2 \tag{1}$$

where $\bar{\omega} = \omega h/(\pi c_T)$, $\bar{k} = kh/\pi$ and $c_T = \sqrt{\mu/\rho}$ is the shear velocity, $m = 0, 2, 4, \ldots$ for symmetric wave modes, and $m = 1, 3, 5, \ldots$ for antisymmetric wave modes. The group velocity of *SH* guided wave can be readily obtained as

$$\bar{c}_g = \sqrt{1 - \left(\frac{m}{\bar{\omega}}\right)^2} \tag{2}$$

The cutoff frequency can be readily derived by setting $\bar{k} = 0$, leading to

$$\bar{\omega}_c = m \tag{3}$$

The displacements for *SH* waves for each *m*th mode can be derived as

$$u_m(x, z, t) = a_m v_m(z) e^{i(k_m x - \omega t)} \tag{4}$$

where a_m is the amplitude and

$$v_m(z) = \cos(m\pi\bar{z} + \gamma) \tag{5}$$

where $\gamma = 0$ or $\pi/2$ represents symmetric or antisymmetric wave modes, respectively. And $\bar{z} = z/h$.

The two shear stress components for each *m*th mode can be obtained as

$$\begin{bmatrix} \tau_x^m \\ \tau_z^m \end{bmatrix} = a_m \mu \begin{bmatrix} ik_m \\ q_m \end{bmatrix} v_m(z) e^{i(k_m x - \omega t)} \tag{6}$$

and $q_m^2 = (\omega/c_T)^2 - k_m^2$.

2.2. Reciprocity relation and power flow

The complex reciprocity relation for time-harmonic functions in integral form is given as [21]

$$\int_s (\mathbf{v}_2^* \cdot \boldsymbol{\sigma}_1 + \mathbf{v}_1 \cdot \boldsymbol{\sigma}_2^*) \cdot \mathbf{n} dS = -\int_V (\mathbf{v}_2^* \cdot \mathbf{f}_1 + \mathbf{v}_1 \cdot \mathbf{f}_2^*) dV \tag{7}$$

where $\mathbf{n}$ is the surface outward normal, $[\mathbf{v}_1, \boldsymbol{\sigma}_1, \mathbf{f}_1]$ and $[\mathbf{v}_2, \boldsymbol{\sigma}_2, \mathbf{f}_2]$ are velocities, stresses and body forces for wavefields 1 and 2, * denotes the complex conjugate.

The domain for applying the *complex reciprocity relations* (Eq. (7)) for both wavefields is considered from incident input field, say $x = 0$ to an arbitrary x, $-h/2 \le z \le h/2$. Since the top and bottom surfaces ($z = \pm h/2$) of the plate are traction-free, the surface integrations at $z = \pm h/2$ in the complex reciprocity relation disappear. After substituting displacements and stresses components for *SH* waves into Eq. (7), and setting the body force terms to be zero, yields the following simplified relation

$$4P_{mn}[e^{i(k_m - k_n^*)x} - 1] = 0 \tag{8}$$

where

$$P_{mn} = -\frac{1}{4}\int_{-h/2}^{h/2} [\mathbf{v}_n^*(z) \cdot \boldsymbol{\sigma}_m(z) + \mathbf{v}_m(z) \cdot \boldsymbol{\sigma}_n^*(z)] \cdot \mathbf{n}_x dz \tag{9}$$

The orthogonality relation gives

$$P_{mn} = 0 \quad (k_m \neq k_n^*) \tag{10}$$

For a given *SH* mode passing through any arbitrary position *x*, the power flow through the surface per unit width can be given by [21]

$$P_m = -\frac{1}{2}\int_{-h/2}^{h/2} [\mathbf{v}_m^*(z) \cdot \boldsymbol{\sigma}_m(z)] \cdot \mathbf{n}_x dz \tag{11}$$

By letting the *n*th mode in Eq. (8) satisfying $k_m = k_n^*$ to be m^*, and combining Eq. (7), Eq. (11) can be written as

$$P_m = P_{mm^*} \tag{12}$$

If the *m*th mode is a propagating mode with real wavenumbers, then $m^* = m$, which indicates that the propagating mode *m* radiating energy by interacting only with itself but not with any other modes. If the *m*th mode is an evanescent mode with pure imaginary wavenumber, *i.e.*, $k_m = k_{m^*}^*$, the non-propagating modes *m* and m^* decay in the $\pm x$ directions, respectively. This implies that evanescent *SH* mode can only transport energy when it interacts with an evanescent mode decaying in the opposite direction.

Therefore, for *propagating SH wave*, the power flow passing through any arbitrary position *x* can be determined analytically from Eq. (11) as

$$P_m = \frac{1}{4}\mu\omega h k_r^m |a_m|^2 = \frac{1}{4}\rho\omega^2 c_g^{(m)} h |a_m|^2 \tag{13}$$

where k_r^m denotes real wavenumber. Similarly, the power flow for *evanescent SH wave*

$$P_m = \frac{1}{4} i\mu\omega h k_i^m |a_m|^2 e^{-2k_i^m x} \tag{14}$$

where k_i^m represents imaginary wavenumber. From Eq. (14), it is obvious that the power flow for evanescent *SH* modes is location dependent and exponentially decays as it propagates out. Although evanescent *SH* mode itself cannot transport energy, the pure imaginary power flow stands for the reactive power which cannot be consumed and is stored in the plate.

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