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Finite element analysis of the Rayleigh wave scattering in isotropic bi-material wedge structures

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ABSTRACT

The numerical study is performed of the harmonic Rayleigh wave scattering in a composite structure constructed from two elastically isotropic 90°-wedges. These wedges are in contact along one pair of their faces. It is assumed that either the perfectly sliding contact or the perfectly rigid one is realized. The other pair of faces forms a plane border between the resulting bi-material wedge and the exterior half-infinite space occupied by vacuum. The finite element method is used. The perfectly matched layer spatially confines the computational domain. The dependences of the reflection and transmission coefficients of the Rayleigh wave on the angle of incidence, the Poisson ratio and the type of contact are obtained and analyzed for different combinations of materials. The behavior of the coefficient of the Rayleigh wave conversion into the interfacial wave which may exist on the internal boundary of the structure is also investigated. A number of relations between the coefficients of conversion are derived from symmetry considerations for structures with sliding contact and composed of identical isotropic materials.

1. Introduction

The study of the elastic wave reflection and refraction at the interface between two media is one of the classical areas of physical acoustics. Currently comprehensive theoretical and experimental investigations are performed of bulk wave reflection/refraction phenomena [1–5]. On the one hand, such an interest is stipulated by the fact that it is the bulk wave transmission across interfaces that one has to deal with in practice most often, in particular, when employing the ultrasound for non-destructive testing of materials [6]. On the other hand, a significant progress in the understanding of basic properties of the bulk wave reflection/refraction is due to a relative simplicity of solving relevant boundary-value problems. First of all, we bear in mind derivation and analysis of the reflection and transmission coefficients for plane harmonic waves at plane interfaces between half-infinite media or plates [1–6]. These coefficients can be derived analytically in elastically isotropic media. In crystals, the anisotropy commonly does not permit one to calculate explicitly the conversion coefficients in terms of the material constants and the angle of incidence, except for certain symmetric geometries of propagation. Nevertheless it appears possible to analyze the behavior of these coefficients in the neighborhood of some critical angles of incidence, such as angles corresponding to the excitation of leaky waves as well as at quasi normal incidence in piezoelectrics [7–14]. The particularity of the latter case is related with the occurrence of quasi-static electric fields.

If analytical solutions are not available, then the conversion coefficients can be computed numerically in a fairly simple way. Finding solutions of bulk wave propagation problems in layered structures markedly simplifies when employing the so-called transfer-matrix in combination with special methods which allow the removal of instabilities occurring in numerical computations when the layer thickness exceeds a few wavelengths [15–22]. In some cases the use of transfer-matrix methods offers a possibility of transparently deriving and analyzing analytic expressions for the bulk wave reflection and transmission coefficients in layered media, e.g. [23–26].

The situation is quite different regarding surface acoustic waves. Any considerations of the surface wave transmission from one surface to the neighboring surface of the same substrate or from one medium to another reduce to solving surface wave scattering problems in homogeneous or inhomogeneous,

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e.g., bi-material, wedges and wedgelike structures. This class of boundary-value problems does not admit fully analytical solutions even for isotropic solids. However, approximate analytic expressions of the reflection and transmission coefficients are obtained for isotropic homogeneous wedges [27–29]. The generation of bulk waves is either totally neglected or not fully taken into account. Under similar assumption approximate expressions are also derived for the shear polarized Bleustein-Gulyaev wave in a 90° piezoelectric wedge of symmetry 6 mm [30,31].

There are several methods of solving exactly the Rayleigh wave scattering problem in isotropic wedges. One of them uses the free-space Green's function for displacement fields [32–37]. After a chain of involved evaluations a set of integral equations is obtained and solved numerically in order to find the reflection coefficient from the wedge apex and the transmission coefficient determining the amplitude of the Rayleigh wave transmitted on the second face of the wedge. The second method is based on the Sommerfeld-Malyuzhinets technique [38–43]. The initial boundary-value problem is eventually reduced to an integral equation which again is solved numerically. The reflection and transmission coefficients can also be found by the boundary element method [44].

The above-listed three approaches imply preliminary transformations of the wave equations. A different group of methods consists in numerically solving directly the boundary-value problem without any transformations of the wave equations. Following such a line the reflection and transmission coefficients in isotropic homogeneous wedges were computed by the finite difference method and by a mixed finite element/finite difference method [45,46].

In our paper [47] we simulated the Rayleigh wave scattering in isotropic 90° homogeneous wedges using the finite element method (FEM). It was assumed that the Rayleigh wave is harmonic, i.e. computations were performed in frequency domain. The computational domain was spatially confined by the so-called perfectly matched layer (PML) [48–53]. Our main interest was in investigating the scattering at the rounded apex of the wedge as well as in the case where a thin layer of foreign material is deposited on the face which scatters the Rayleigh wave. However, in order to validate our approach, we also computed the reflection and transmission coefficients in wedges with 90° apex angle without any rounding, since it is the case studied in Refs. [32–46]. Our results are in a good agreement with the results obtained in those works.

Subsequently we investigated by FEM the scattering of harmonic surface wave from 90°-edges of anisotropic piezoelectric substrates [54]. In particular, it was shown that in anisotropic substrate surface waves are generally scattered off differently at the right-hand border and the left-hand border. The difference between the conversion coefficients can be significant.

The surface wave scattering in a single homogeneous wedge with free faces can be viewed as a counterpart of the bulk wave reflection from the mechanically free boundary of half-infinite substrates. The bulk wave reflection at the interface between two elastic media can be associated with the surface wave scattering in a inhomogeneous wedge composed of two different wedges by joining them along a pair of faces. The Rayleigh wave scattering in bi-material isotropic wedges has earlier been studied in Refs. [55,56] on the basis of the Sommerfeld-Malyuzhinets technique. The normal incidence on the wedge apex is considered. From the solution of the appropriate integral equations the dependence is obtained of the conversion coefficients, including the coefficient of excitation of the Stoneley wave, on the value of the opening angles of each wedges constituting the composite structure.

In the present paper we model by FEM the Rayleigh wave scattering in a bi-material wedge constructed of two 90° elastically isotropic wedges. We want to investigate the dependence of the conversion coefficients on the angle of incidence of the Rayleigh wave and on the existence or non-existence of an interfacial wave at the interface between the wedges. Two types of the contact at the interwedge border will be considered, namely, the perfect rigid and the perfect sliding contacts. We also repeat some of computations carried out in Ref. [56] (see Appendix).

2. Statement of the problem

Let an inhomogeneous substrate be composed of two elastically isotropic quarter-spaces (domains 1 and 2 in Fig. 1). These quarter-spaces are in an acoustic contact along their vertical faces. Above the substrate is vacuum. The harmonic Rayleigh wave

$$\mathbf{u}_{in}(\mathbf{r},t) = \mathbf{A}_{in}(z)e^{ik_{in}(x\cos\theta + y\sin\theta - v_{in}t)}, \tag{1}$$

is incident from domain 1 on domain 2 along the direction making an angle θ with the coordinate x-axis. The symbols k_{in} and v_{in} stand for the wavenumber and the velocity, respectively. The vector $\mathbf{A}_{in}(z)$ specifies the dependence of the Rayleigh wave displacement on the z-coordinate.

At the border between domains 1 and 2 the incident wave transforms into the reflected Rayleigh wave propagating in domain 1 and into the transmitted Rayleigh wave propagating on the horizontal surface of domain 2. In analogy with bulk waves, the transmitted wave occurs if the angle θ is smaller than the critical angle $\theta_o = \arcsin(v_{in}/v_{tr})$, where v_{tr} is the Rayleigh wave velocity in domain 2. In addition, bulk waves are generated and an interfacial wave can occur at the interwedge border. This interfacial wave may or may not exist depending on the material constants and the type of the contact.

We assume that the contact is either perfectly rigid or perfectly sliding. It can be proved that not more than 1 interfacial wave exists on the rigid contact and not more than 2 slip waves exist on the sliding contact between two solids not possessing piezoelectric or piezomagnetic properties, the second slip wave occurring only in the case of anisotropic media[1][57–59].

Our aim is to compute the reflection coefficient R, the transmission coefficient T, and the coefficient of conversion into the interfacial wave T_{iaw}. These coefficients are defined as the ratios of the normal displacement of the appropriate wave at the relevant boundary to the z-component $A_{in,z}$ of the displacement $\mathbf{A}_{in}$ ($|\mathbf{A}_{in}| = 1$) of the Rayleigh wave at the boundary $z = 0$.

3. General description of the solution procedure

First of all the scattered fields $\mathbf{u}_{sc}(\mathbf{r},t)$ and $\mathbf{w}_{sc}(\mathbf{r},t)$ in domains 1 and 2, respectively, are to be found by FEM.[2] The FEM functional is written in such a way that it corresponds to the wave equations for the total displacement $\mathbf{u}_{tot} = \mathbf{u}_{sc} + \mathbf{u}_{in}$ and the displacement $\mathbf{w}_{sc}$ in domain 1 and 2, respectively:

$$\nabla \cdot \hat{\boldsymbol{\sigma}}_{tot} = -\rho^{(1)}\omega^2\mathbf{u}_{tot}, \tag{2}$$

$$\nabla \cdot \hat{\boldsymbol{\sigma}}_{w} = -\rho^{(2)}\omega^2\mathbf{w}_{sc}, \tag{3}$$

where $\omega = v_{in}k_{in}$ is the frequency,

$$\hat{\boldsymbol{\sigma}}_{tot} = \hat{\mathbf{c}}^{(1)}\nabla \cdot \mathbf{u}_{tot}, \quad \hat{\boldsymbol{\sigma}}_{w} = \hat{\mathbf{c}}^{(2)}\nabla \cdot \mathbf{w}_{sc} \tag{4}$$

are the mechanical stress tensors, $\hat{\mathbf{c}}^{(1,2)}$ and $\rho^{(1,2)}$ are the elastic moduli and the density of domain 1 and 2, respectively. Since all fields depend on the y-coordinate as $\exp[ik_{in}y\sin\theta]$, the space derivative $\partial/\partial y$ is replaced with $ik_{in}\sin\theta$. Therefore only the depen-

[1] At most 2 interfacial waves exist at the rigid contact and at most 3 waves exist at the sliding contact if the solids are piezoelectrics (piezomagnetics) or possess simultaneously piezoelectric and piezomagnetic properties [60–63]. A vacuum gap separating two piezoactive crystals allows 4 branches of interfacial waves [61].

[2] Our program is written on the basis of the COMSOL Multiphysics and MATLAB packages.