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Frequency and wavelength prediction of ultrasonic induced liquid surface waves

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ABSTRACT

A theoretical investigation of parametric excitation of liquid free surface by a high frequency sound wave is preformed, using potential flow theory. Pressure and velocity distributions, resembling the sound wave, are applied to the free surface of the liquid. It is found that for impinging wave two distinct capillary frequencies will be excited: One of them is the same as the frequency of the sound wave, and the other is equal to the natural frequency corresponding to a wavenumber equal to the horizontal wavenumber of the sound wave. When the wave propagates in vertical direction, mathematical formulation leads to an equation, which has resonance frequency equal to half of the excitation frequency. This can explain an important contradiction between the frequency and the wavelength of capillary waves in the two cases of normal and inclined interaction of the sound wave and the free surface of the liquid.

1. Introduction

Study of capillary waves is an important subject due to their role in liquid atomization. Atomization takes place by high frequency capillary waves and their instability. The produced particle size is related to these waves characteristics. Inducing capillary waves by vibrating surfaces, ultrasonic transducers and Surface Acoustic Wave (SAW) are among the reported methods for atomization.

This paper concerns with capillary waves generated by either normal or inclined interaction of the sound wave with liquid free surface. Application of the sound wave for atomization of liquids is known from the experimental work of Wood and Loomis [1]. In this process, known as Ultrasonic Atomization (UA), capillary waves are generated by an ultrasonic transducer inside the liquid with its axis normal to the free surface. Instability of the capillary waves and cavitation near the free surface are the two main phenomena responsible for atomization [2,3], however, in low frequencies (20–100 kHz) the former is dominant [3].

Surface waves induced by vertical propagating sound waves are shown to be similar to Faraday waves generated on the surface of a liquid inside a vertically oscillating container [4]. Therefore the frequency of these waves are expected to be one half of the excitation frequency. Benjamin and Ursell investigated Faraday waves by neglecting the effect of viscosity and assuming small amplitude fluctuations in the surface [5]. Their formulation finally led to the well-known Mathieu equation. This equation implies subharmonics of frequencies $f/2,\ 3f/2,\ \ldots$, and harmonics of $f,\ 2f,\ \ldots$, in response to an excitation frequency of f, where only subharmonic of $f/2$ is excited for small amplitude waves [5].

Eisenmenger [6] showed that the frequency of capillary waves in UA is also equal to half of the excitation sound frequency. Lang measured the capillary wavelength and resulting atomized particles size produced by UA [7]. He obtained the following correlation between droplets mean diameter, D, and capillary wavelength, λ_c

$$D = \alpha\lambda_c, \tag{1}$$

where $\alpha = 0.34$. Employing Kelvin relation for capillary wavelength [8] and Eisenmenger result, Lang suggested [7]

$$D = \alpha\left(\frac{8\pi\sigma}{\rho f^2}\right)^{\frac{1}{3}}. \tag{2}$$

where ρ and σ are density and surface tension of the liquid, respectively.

A number of theoretical studies on the subject of capillary waves, induced by ultrasonic transducer have been reported in the literature [9–12]. They considered a thin layer of liquid film and modeled the ultrasonic excitation as vertical vibration of this film. The resulting formulation, that lead to Mathieu equation, are similar to Benjamin and Ursell [5] model for Faraday waves.

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Isseman et al. performed numerical and analytical investigation of the mentioned problem, but because of the time averaging and ignoring of the effect of vertical displacement of the free on spatial phase of sound wave, they observed no instability [13].

Liquid atomization by SAW was introduced by Kurosawa [14]. In this method an ultrasonic Rayleigh wave, which is generated on the surface of a piezoelectric substrate, is transfered with Rayleigh angle into the liquid volume placed on top of it. Unlike the UA, here the sound wave propagation angle does not make a right angle with the mean level of free surface. Kurosawa assumed that a relation, similar to Lang's relation, is applicable for correlating his measurements, though his interacting wave was not vertical. He suggested $\alpha = 3.8$ which is one order of the magnitude greater than the Lang's finding of 0.34. Similar relation was assumed by others [15–19] to correlate the measurements on the diameter of atomized particles produced by SAW, but the discrepancy with their observations was large.

Experimental reports of the capillary waves generated by SAW in the liquid droplet surface have two conclusions: first, the existence two widely spaced important frequency regions in the low and high frequency range, and second observation of large wave amplitude at lower frequencies compared to that of higher frequencies [16,17,20,21]. In these works, for a 20 MHz excitation frequency, waves with frequencies as low as 0.1–10 kHz and also with a high frequency of 20 MHz were observed, with no evidence of the $f/2$ frequency. This suggested that a formula similar to Lang's, which has its root in vertical excitation of liquid free surface, was not suited to correlate the experimental findings in this situation.

To resolve this discrepancy Qi et al., who gave the first experimental frequency spectrum of these capillary waves, argued that this is due to the very high SAW applied frequencies [16]. They used scale analysis and wrote two balance equations between viscous and capillary forces as [22]

$$f_c \sim \frac{\sigma}{\mu R}, \tag{3}$$

and between inertial and capillary forces as [22]

$$f_c \sim \left(\frac{\sigma}{\rho R^3}\right)^{1/2}, \tag{4}$$

where R is a characteristic length. Other authors [17,20,21,18,23,24] also considered this relation to predict the frequency of capillary waves instead of the $f/2$ suggestion of Lang [7].

In addition to these frequency observations, Alvarez et al. [18] and Li et al. [24] measured the wavelength of capillary waves on a film of water induced by SAW. They found that capillary wavelength is near to the SAW wavelength. Alvarez et al. reported the experiment for Bovine Serum Albumin (BSA) and found similar results in spite of the change in liquid properties.

Theoretical study of capillary waves induced by inclined impinging sound wave are scarce. Qi et al. [16] and Collins et al. [25] studied these waves by neglecting inertia in the vertical direction, although it could be noticeable due to the exponential variation of vertical velocity near the free surface; and very low time scale. The most significant work was performed by Tan et al., where by using the method of multiple scales, the coupled equations of fluid and substrate was solved numerically [26]. Their simulation well predicted the wavelength.

In this paper, a single formulation is derived for the liquid motion induced by either an inclined or a vertical propagating plane wave. It will be shown that horizontal phase difference of the sound velocity on the free surface, in case of inclined propagating wave, causes the growth of capillary waves with a wavelength equal to the horizontal wavelength of the sound, unlike normal propagating sound, for which the forcing frequency specifies the wavelength. This fact is illustrated using potential flow theory, which suggests that viscosity does not play an important role in the frequency of the generated waves.

2. Problem statement

The motion of the free surface of a liquid, excited by a plane sound wave, is studied theoretically, considering inviscid, incompressible two dimensional flow inside a semi-infinite container with finite depth with a flat horizontal bottom (Fig. 1). The x and z axes are defined with horizontal and vertical directions, respectively, and the equation for the shape of the free surface is

$$z = \eta(x, t), \tag{5}$$

where η denotes the elevation of the surface above its reference level ($z = 0$). The amplitude of oscillations of the free surface is assumed to be small with respect to the wavelength and equations are linearized around $z = 0$. The gas above the container is considered to have negligible density and viscosity relative to the liquid.

The incident plane wave (p_i) propagates with angle θ_i, and scatters from the interface (Fig. 1). Linear wave theory is used to find the pressure and velocity distributions on the free surface, the effect of the average flow field inside liquid volume on the sound propagation is neglected and the two fluids are assumed to have large acoustic impedance difference (e.g. gas/water). Assuming low amplitude vibrations of the free surface, the reflected and the transmitted waves are considered to be planar.

2.1. Governing equations

Liquid velocity potential $\boldsymbol{V}_l = \nabla\phi(x, t)$ should satisfy Laplace equation

$$\nabla^2\phi = 0, \tag{6}$$

with Neumann boundary condition on the bottom

$$\phi_z = 0 \quad at \quad z = -h, \tag{7}$$

and the linearized free surface kinematic and dynamic conditions, which respectively are

$$\eta_t = \frac{\partial\phi}{\partial z} + v_{z,ac} \quad at \quad z = 0, \tag{8}$$

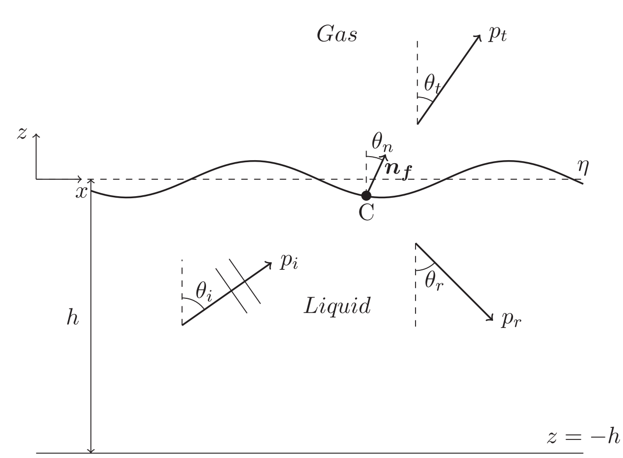

Fig. 1. The schematic view of a semi-infinite liquid volume excited by an incident sound wave p_i. The incident sound wave reaches the surface with angle θ_i with respect to vertical z axis, and reflects by the angle θ_r. Part of the incident wave transports to the gas (p_t), with angle θ_t with respect to the vertical axis.