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Laser-generated ultrasonic pulse shapes at solid wedges

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ABSTRACT

Laser pulses focused near the tip of an elastic wedge generate acoustic waves guided at its apex. The shapes of the acoustic wedge wave pulses depend on the energy and the profile of the exciting laser pulse and on the anisotropy of the elastic medium the wedge is made of. Expressions for the acoustic pulse shapes have been derived in terms of the modal displacement fields of wedge waves for laser excitation in the thermo-elastic regime and for excitation via a pressure pulse exerted on the surface. The physical quantity considered is the local inclination of a surface of the wedge, which is measured optically by laser-probe-beam deflection. Experimental results on pulse shapes in the thermo-elastic regime are presented and confirmed by numerical calculations. They pertain to an isotropic sharp-angle wedge with two wedge-wave branches and to a non-reciprocity phenomenon at rectangular silicon edges.

1. Introduction

Laser ultrasonics is a viable tool for non-destructive investigation and characterisation of solid surfaces, thin films etc. [1,2]. Guided acoustic waves play a central role in this field. On the one hand, they are generated as a vehicle to detect or image surface defects or extract elastic properties of films, for example. On the other hand, laser ultrasonics is used to study properties of guided acoustic waves themselves. In particular, this experimental technique has been applied to wedge acoustic waves by several researchers (for a recent review see [2]) [3–6]. Acoustic wedge waves are guided by the straight edge of an elastic solid [7–9] and have a number of interesting properties different from surface acoustic waves. Especially, they do not suffer diffraction since they are one-dimensionally guided waves. Sharp-angle wedges are multi-mode systems.

The majority of the experimental studies carried out on wedge waves were concerned with dispersion due to modifications of the ideal wedge tip, e.g., coating of a surface, mostly for isotropic materials (for example, see the reviews [2,10]). The only study known to us focusing on wedge acoustic pulse shapes resulting from thermo-elastic excitation of dispersive wedge waves is [11]. Only recently, effects of anisotropy on the propagation properties of wedge waves in wedges with high-quality tips have been addressed in experiments. Acoustic waves guided at the perfect tip of a homogeneous elastic wedge have in common with surface acoustic waves, propagating at the planar surface of a homogeneous elastic medium, the property of being non-dispersive. In a perfect elastic wedge, the waveforms of acoustic wedge waves do not change during propagation apart from effects of nonlinearity and damping, which are largely negligible in single-crystal silicon over the propagation distances usually studied in laser-ultrasonics experiments if the strains associated with the acoustic waves are sufficiently small.

Anisotropy leads to a number of interesting phenomena in connection with acoustic wedge waves like strongly tip-localized leaky wedge waves [6,12], for example. Here, we are concerned with the pulse shapes of acoustic wedge waves generated in laser-based experiments by short laser pulses focused on one of the two surfaces of an anisotropic wedge. The notion of laser-generated pulse shapes is needed if these wedge pulses shall be used in non-destructive evaluation or in studies of nonlinear effects at crystal edges, where changes of the pulse shapes due to these effects are observed. Also, a comparison is made between pulse shapes of surface acoustic waves and wedge acoustic waves.

In order to counteract diffraction of surface acoustic waves, SAW pulses are often excited via a line source, realized by focusing the laser pulse on the surface by a cylindrical lens. Wedge waves are one-dimensionally guided waves and therefore non-diffractive. Consequently, the laser pulse may be focused on a spot on one of the two surfaces at the tip of the wedge without losing intensity through diffraction. It is shown how the acoustic pulse shape varies with the shape of the excitation spot, especially on its extension in the direction vertical to the apex line. In addition,

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we study the pulse shapes of wedge waves in sharp-angle isotropic wedges, where more than one mode exists for a given frequency.

In the following section, general expressions are derived for laser-excited acoustic pulse shapes in terms of the modal displacement field associated with the guided acoustic waves. In the derivation of these expressions for the thermo-elastic regime, approximations are used which hold for the silicon and silica samples investigated experimentally, where the penetration depth of laser radiation into the medium is small in comparison to the penetration depths of the acoustic waves [13]. The agreement between experimental and theoretical pulse shapes additionally confirms these approximations. Apart from the thermo-elastic regime of laser excitation, we consider excitation of acoustic pulses via a spatially localized short pressure pulse exerted on one of the two surfaces of an elastic wedge. This excitation mechanism was invoked as a simple model for laser-based generation of surface acoustic pulses in a regime, where the acoustic waves are excited predominantly via ablative momentum transfer [14–18]. Here, care has to be taken when applying this linear approximation to the highly complex and nonlinear processes occurring at the surface at laser pulse energies that lead to ablation. Nevertheless, pulse shapes for surface acoustic waves calculated within this approximation showed good agreement with corresponding pulse shapes determined experimentally. Section 2 starts with this latter case, since the theoretical analysis is slightly easier here than in the thermo-elastic regime.

The general expressions derived in Section 2 may be evaluated numerically with various methods, including the finite element method. In Section 3, it will be shown how they are evaluated by using an expansion of the displacement field of wedge waves in a double series of Laguerre functions [8,19–21].

Results will be presented for the displacement gradient at the surface that is measured with the probe-beam deflection technique. Measurements were performed in the thermo-elastic regime for isotropic silica wedges and for several wedge configurations made of anisotropic silicon crystals. Their results are discussed and compared with calculations based on the above-mentioned expressions.

2. General theory

In the following, we consider elastic wedges with arbitrary wedge angle θ and choose the coordinate system as shown in Fig. 1a. The apex line is along the x_1-direction. The x_3-axis is normal to the surface on which acoustic waves are excited and detected. It points into the medium. With few straightforward modifications, the case of the x_2-axis (instead of the x_3-axis) being normal to this surface is obtained, too.

2.1. Pulse shapes generated by a surface pressure pulse

We first consider the case of acoustic wave excitation by a space- and time-dependent compressive stress $p(x_1, x_2, t)$. This leads to the boundary condition

$$\sigma_{\alpha 3}(x_1, x_2, 0, t) = -\delta_{\alpha 3} p(x_1, x_2, t) \tag{2.1}$$

for the stress tensor $(\sigma_{\alpha\beta})$, which is related to the displacement gradients $u_{\alpha,\beta} = \partial u_\alpha / \partial x_\beta$ via the constitutive equation

$$\sigma_{\alpha\beta} = C_{\alpha\beta\mu\nu} u_{\mu,\nu}. \tag{2.2}$$

Throughout this paper, lower-case Greek letters denote Cartesian indices, and summation over repeated Cartesian indices is implied. The displacement field $u_\alpha(x_1, x_2, x_3, t)$ has to satisfy the equation of motion

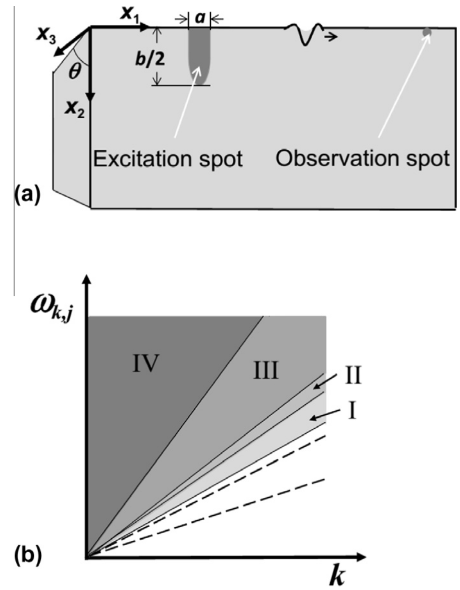

Fig. 1. (a) Geometry and definition of the coordinate system. (b) Acoustic mode spectrum of an infinite wedge, schematical. Dashed lines: wedge wave branches, I: surface waves on one surface, II: surface waves on both surfaces, III: surface and quasi-shear bulk waves, IV: surface and bulk waves (quasi-shear and quasi-longitudinal).

$$\rho \frac{\partial^2}{\partial t^2} u_\alpha = \frac{\partial}{\partial x_\beta} \sigma_{\alpha\beta} \tag{2.3}$$

along with the boundary condition (2.1). Here ρ is the mass density of the medium.

The second surface of the wedge is traction-free. The determination of the displacement field is essentially analogous to the solution of the Lamb problem for an elastic wedge. We start from the variational principle [22]

$$\delta \int_{t_1}^{t_2} dt \left\{ \int_V d^3x \left(\frac{1}{2} \rho \dot{u}_\alpha \dot{u}_\alpha - \frac{1}{2} u_{\alpha,\beta} C_{\alpha\beta\mu\nu} u_{\mu,\nu} \right) - \int_S dS\, p u_3 \right\} = 0. \tag{2.4}$$

In (2.4), the first integral is over the volume V of the infinite wedge, the second is an integral over the surface S of the wedge on which the pressure p is exerted. (dS is the surface element.)

The displacement field is written as a linear combination of the vibrational normal modes of the wedge,

$$u_\alpha(x_1, x_2, x_3, t) = 2\mathrm{Re} \int_0^\infty \left\{ \sum_j U_\alpha(x_2, x_3; k, j)\, A(k, j; t) + \int U_\alpha(x_2, x_3; k, j) A(k, j; t) dj \right\} e^{ikx_1} \frac{dk}{2\pi}. \tag{2.5}$$

The first term in the curly bracket of (2.5) refers to the discrete part, the second term to the continuous part of the spectrum, which corresponds to the solution of an eigenvalue problem for fixed (one-dimensional) wave-vector k along the tip of the wedge. This eigenvalue problem results from (2.3) and (2.2) for traction-free surfaces and fixed wavenumber, if a sinusoidal time dependence of the displacement field is assumed (see [23,24] for the isotropic case). The corresponding eigenvalues $\omega_{k,j}^2$ are the squares of the frequencies of the normal modes. Here, j labels the normal modes having the same wave-vector k parallel to the tip of the edge. $A(k, j; t)$ are yet unknown amplitudes, and $U_\alpha(x_2, x_3; k, j)$ are modal functions normalized in an appropriate way. In the case of an infinite elastic wedge considered here, the discrete part of the spectrum is associated with wedge waves having displacements