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Dispersion relations of elastic waves in one-dimensional piezoelectric/piezomagnetic phononic crystal with functionally graded interlayers

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ABSTRACT

The effects of functionally graded interlayers on dispersion relations of elastic waves in a one-dimensional piezoelectric/piezomagnetic phononic crystal are studied in this paper. First, the state transfer equation of the functionally graded interlayer is derived from the motion equation by the reduction of order (from second order to first order). The transfer matrix of the functionally graded interlayer is obtained by solving the state transfer equation with the spatial-varying coefficient. Based on the transfer matrixes of the piezoelectric slab, the piezomagnetic slab and the functionally graded interlayers, the total transfer matrix of a single cell is obtained. Further, the Bloch theorem is used to obtain the resultant dispersion equations of in-plane and anti-plane Bloch waves. The dispersion equations are solved numerically and the numerical results are shown graphically. Five kinds of profiles of functionally graded interlayers between a piezoelectric slab and a piezomagnetic slab are considered. It is shown that the functionally graded interlayers have evident influences on the dispersion curves and the band gaps.

1. Introduction

As a kind of artificial periodic composite materials or structures, phononic crystal is of characteristic of the band gap. Due to the convenience of adjusting band gap, the phononic crystal consisting of piezoelectric materials and piezomagnetic materials has attracted attentions of many researchers in recent years. Alvarez-Mesquida et al. [1] discussed the shear horizontal wave in layered piezoelectric composites in terms of a recursive system of equations involving the piezoelectric impedance. Qian [16,17] studied the dispersion relation of the SH-waves in a periodic piezoelectric-polymeric layered structure and the influences of initial stress on the stop bands and pass bands of anti-plane waves. Monsivais et al. [12] studied the surface and shear horizontal waves in a piezoelectric composite media consisting of piezoelectric layers of hexagonal 6 mm symmetry. It is considered that the finite systems including Fibonacci sequences, the systems with a linear perturbation in the piezoelectric parameters and the periodic systems. Chen et al. [5,6] considered harmonic waves propagating in magneto-electro-elastic multilayered plates made of orthotropic elastic (graphite–epoxy), transversely isotropic, piezoelectric and magnetostrictive materials. In the former papers, dispersion relation and the model shape of wave propagation along multilayered plate with traction-free surfaces at top and bottom was studied. In the latter paper, the reflection and transmission coefficients through the multilayered plate were calculated. Pang et al. [14,15] studied further the dispersion relations of Lamb waves and SH waves in layered periodic composites consisting of piezoelectric and piezomagnetic phases. The elastic wave propagation in two-dimensional and three-dimensional phononic crystals with piezoelectric and piezomagnetic inclusions was investigated by Wang et al. [20,21]. In their investigation, the magneto-electro-elastic coupling effects and the initial stress effects were taken into account. The band gap characteristics for three kinds of lattice arrangements were investigated by the plane wave expansion (PWE) method. Sun et al. [18] studied the propagation of SH wave in a cylindrically multiferroic composite consisting of a piezoelectric layer and a piezomagnetic central cylinder in which the interface was damaged mechanically, magnetically or electrically. The dispersion relations of SH wave were obtained for two kinds of electric–magnetic boundary conditions at the free surface. Lan and Wei [10] studied the influence of imperfect interfaces, which was modeled as thin membranes with elasticity and inertial even but without thickness, on the dispersion characteristics and the band gaps of SH waves propagating through a laminated piezoelectric phononic crystal.

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In the investigations mentioned above, only the homogeneous material or the piece-wise homogenous material is involved. The wave propagation in the functionally graded material with material composition and properties varying continuously in certain directions has not been included. Due to the spatial-varying of material constants, the wave propagation in the functionally graded slab is more complicated than in the homogeneous slab. Wang and Rokhlin [19] presented the differential equations governing the transfer and stiffness matrices for a functionally graded generally anisotropic magneto-electro-elastic medium. The wave propagation solution for a thick layer or a multilayered structure of inhomogeneous layers was obtained recursively from the thin layer solutions. Pan and Han [13] presented an exact solution for the multilayered rectangular plate made of functionally graded, anisotropic, and linear magneto-electro-elastic materials. The functionally graded material was assumed to be exponential varying in the thickness direction and the homogeneous solution in each layer was obtained based on the pseudo-Stroh formalism. Wu et al. [23] studied the propagation of elastic waves in one-dimensional (1D) phononic crystals (PCs) with functionally graded materials (FGMs) using the spectral finite elements and transfer matrix methods. Cai and Wei [4] investigated propagation characteristics of elastic waves in two-dimensional (2D) phononic crystal consisting of parallel cylinders or cylindrical shells with varying material parameters along the radial direction embedded periodically in a homogeneous host. The influences of the graded medium with different gradient profiles upon dispersion curves and the band gaps were discussed. Golub et al. [8] discussed time-harmonic plane elastic SH-waves propagating in periodically laminated composites with functionally graded (FG) interlayers. A finite stack of periodic layers between two identical elastic half-planes was considered. The functionally graded (FG) interlayers was treated by two different models, i.e. the explicit FG model and the multilayer model. The reflection and transmission coefficients and band gaps of the periodic laminates are calculated. Fomenko et al. [7] considered the in-plane wave propagation in layered phononic crystals composed of functionally graded interlayers arisen from the solid diffusion of homogeneous isotropic materials of the crystal. Lan and Wei [11] discussed the influences of the graded interlayer with different gradient profiles on the anti-plane elastic wave propagating through a laminated piezoelectric/piezomagnetic phononic crystal. The graded interlayer was modeled as a system of homogenous sublayers with both piezoelectric and piezomagnetic effects simultaneously. The effect of the graded interlayer on the band gap was introduced by inserting an additional transfer matrix of interlayer in the calculation of the total transfer matrix.

In this paper, the one-dimensional phononic crystal composed of the piezoelectric slabs, the piezomagnetic slabs and the functionally graded interlayers are considered. First, the state transfer differential equations in the homogeneous piezoelectrical and pizomagnetical slabs and in the functionally graded interlayer are derived. The transfer matrixes of the homogeneous piezoelectric slab and the pizomagnetical slab are obtained by solving the state transfer equation with constant coefficient while the transfer matrix of the functionally graded interlayer is obtained by solving the state transfer equation with spatial-varying coefficient. Then, the total transfer matrix of one typical single cell of the periodical structure is obtained by the combination of these transfer matrixes. Finally, the Bloch theorem is used to obtain the dispersion equations of Bloch waves. Five kinds of profiles of functionally graded layers between piezoelectric slabs and piezomagnetic slabs are considered. The dispersion equations of in-plane Bloch waves and anti-plane Bloch waves are both solved and the numerical results are shown graphically. Based on these numerical results, the influences of functionally graded interlayers on the dispersion curves and band gaps are discussed.

2. Transfer matrix of coupled waves in each slab

2.1. Linear vector differential equation for the state vector

Consider a one-dimensional phononic crystal which is formed by periodically repeating four different transversely isotropic slabs, i.e. a homogeneous piezoelectric slab (PE), a homogeneous piezomagnetic slab (PM) and two different functionally graded layers ($\mathrm{FGL}^{\mathrm{I}}$ and $\mathrm{FGL}^{\mathrm{II}}$), as shown in Fig. 1. We establish four local Cartesian coordinate systems, which are indicated by the superscripts $\prime$, $\prime\prime$, I' and II', respectively. In latter formulation, if the physical quantity doesn't have any superscript, it will be appropriate for all local Cartesian coordinate systems. Let the x_3-axis is the poling direction and the slab is transversely isotropic in the ox_1x_2 coordinates plane. In each slab, c_{ijmn}, e_{mij}, q_{mij}, ε_{mi}, μ_{mi} and α_{mi} are the elastic, piezoelectric, piezomagnetic, dielectric, magnetic permeability and magnetoelectric parameters, respectively. ρ and d are the mass density and thickness. The piezomagnetic coefficient and the magnetoelectric coefficient are zero ($q'_{mij} = 0$ and $\alpha'_{mi} = 0$) for the piezoelectric slab while the piezoelectric coefficient and the magnetoelectric coefficient are zero ($e''_{mij} = 0$ and $\alpha''_{mi} = 0$) for the piezomagnetic slab. For the piezoelectric slab and the piezomagnetic slab, all material parameters are constant quantities. The material parameters are only functions of only x_3-axis coordinate for two different functionally graded interlayers.

The constitutive equations of the transversely isotropic magnetic-electro-elasto medium are [11]

$$\begin{cases} \sigma_{ij} = c_{ijkl}S_{kl} - e_{mij}E_m - q_{mij}H_m \\ D_m = e_{mij}S_{ij} + \varepsilon_{mi}E_i + \alpha_{mi}H_i \\ B_m = q_{mij}S_{ij} + \alpha_{mi}E_i + \mu_{mi}H_i \end{cases} \tag{1}$$

where σ_{ij} and S_{mn} are the stress and strain tensors, respectively. D_m, E_m, B_m and H_m are the electric displacement, electric field, magnetic induction and magnetic field vectors, respectively. The physical constants of the transversely isotropic piezoelectric slab and piezomagnetic slab are given in Appendix A. The strain tensor S_{mn}, the electric field E_m and the magnetic field H_m are related with the displacement u_n, the electric potential φ and the magnetic potential ψ by

$$S_{mn} = \frac{1}{2}\left(\frac{\partial u_n}{\partial x_m} + \frac{\partial u_m}{\partial x_n}\right), \quad E_m = -\frac{\partial \varphi}{\partial x_m}, \quad H_m = -\frac{\partial \psi}{\partial x_m} \tag{2}$$

in the quasi static electric/magnetic field approximation. The mechanical, electrical and magnetic governing equations can be expressed as

$$\frac{\partial \sigma_{ij}}{\partial x_j} = \rho\frac{\partial^2 u_i}{\partial t^2}, \quad \frac{\partial D_m}{\partial x_m} = 0, \quad \frac{\partial B_m}{\partial x_m} = 0 \tag{3}$$

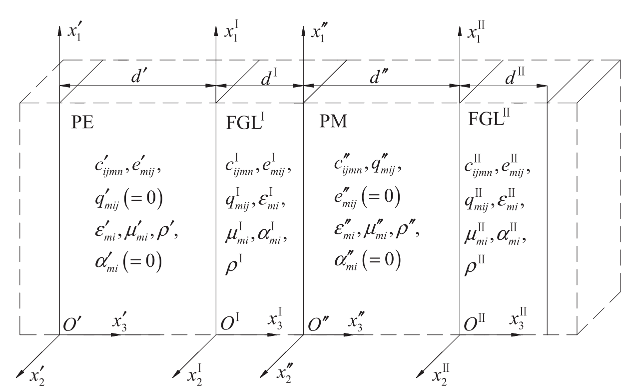

Fig. 1. A typical single cell of one-dimensional piezoelectric/piezomagnetic phononic crystal with functionally graded interlayers.