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Zero-frequency and slow elastic modes in phononic monolayer granular membranes

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ABSTRACT

We theoretically study the dispersion properties of elastic waves in hexagonal and honeycomb monolayer granular membranes with either out-of-plane or in-plane particle motion. The particles interact predominantly via normal and transverse contact rigidities. When rotational degrees of freedom are taken into account, the bending and torsional rigidities of the intergrain contacts can control some of the phononic modes. The existence of zero-frequency modes, zero-group-velocity modes and their transformation into slow propagating phononic modes due to weak bending and torsional intergrain interactions are investigated. We also study the formation and manipulation of Dirac cones and multiple degenerated modes. This could motivate variety of potential applications in elastic waves control by manipulating the contact rigidities in granular phononic crystals.

1. Introduction

There has been a growing interest in investigating the propagation of elastic/acoustic waves in phononic crystals in the last decade [1,2]. Owing to Bragg scattering in spatially periodic media, phononic crystals possess exotic dispersion characteristics including, for example, frequency band gaps [3], negative refraction [4], subwavelength imaging [5], etc. Although studies of phononic crystals involving elastic behaviors have been reported in all dimensions, from 1D to 3D [6–8], most of the recent studies are focusing on different types of two-dimensional lattice that support bulk or edge modes [9–11]. For instance, in the nearly isostatic square/kagome lattices, by accounting for only the nearest neighbor central-force interactions, soft modes and zero-frequency bulk modes have been predicted [12,13]. More interestingly, when the kagome lattices are twisted, negative Poisson ratio and zero-frequency edge states could be achieved [14,15]. Other fascinating elastic properties, such as topological soft modes and topological edge modes, have also been reported in the kagome and honeycomb systems [16,17].

In non-consolidated granular crystals, the interactions between individual grains take place via local contacts, which are much smaller in size than the dimensions of the individual grains and inherently much softer than the grains [18–22], e.g., Hertzian contacts. Even when granular crystals are consolidated via their curing, like opals, or by grain-connecting ligands, like in nanocrystal superlattices, the elastic links between the grains keep being significantly smaller and softer than the grains themselves. This induces propagation of elastic waves in granular structures at significantly slower velocities than in the individual grains [23–25] even if the rotational degrees of freedom of the individual beads are not strongly involved. In contrast to normal forces, which are central forces in most of the spring-mass systems [14–17], in granular crystals the shear forces due to transverse rigidity of the contacts are non-central and can initiate the rotation of the beads. Thus the rotational degrees of freedom of the individual beads, the particle dimensions and the interactions through non-central forces should be taken into consideration. It was theoretically predicted [6,18–20] and experimentally demonstrated [21,22] that, due to the rotational degrees of freedom of the particles, additional coupled rotational/transverse and pure rotational modes can propagate in granular crystals, while the pure transverse modes, predicted by the theoretical case of the frozen grain rotations, are modified into coupled transverse/rotational ones. Interestingly, accounting for the rotational degrees of freedom, in addition to translational ones, can also lead to the existence of zero-frequency (zero-energy) modes [26–28]. The additional rotational degrees of freedom provide extra flexibilities to dispersion engineering of phononic crystals and to the control of the propagation of elastic waves.

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It should be mentioned that, a general theoretical approach for the analysis of acoustic waves in phononic crystals is known already for quite a long time [1–3], and the Cosserat continuum theory for the description of the long-wavelength acoustic wave propagating in the micropolar media with rotational degrees of freedom exists for more than a century [29–31]. In contrast, the analytical discretized models that study the rotational modes and their coupling to other modes, in particular to shear ones, have started to attract increasing interest only in recent years [6,18–20,26,32]. In this work, we exploit the Lagrangian method [33] to evaluate the intergrain interactions of granular phononic membranes. An important advantage of the method is that it is possible to find analytical solutions for modes at high symmetry points, like Γ, M and K. The analytical formulas are very useful by giving clear guidelines on how to control the phonons spectra and to design suitable phononic crystals.

In particular, we theoretically study the dispersion relations of elastic waves in hexagonal and honeycomb monolayer granular membranes for both out-of plane and in-plane motion. We also demonstrate that rotational modes and their coupling to translational modes can provide more flexibilities and additional functionalities in the control of the elastic wave propagation. Specifically, the granular phononic structures are expected to be advantageous in the monitoring of bulk shear and surface Rayleigh acoustic waves [20,34]. Besides, the detailed analysis of zero-frequency modes is reported. In the honeycomb lattices, we demonstrate the existence of zero-group-velocity rotational modes with non-zero frequency, the propagation of which can be initiated by weak bending and torsional interactions between the beads. We also study how the number and parameters of these modes are changing in transition from hexagonal to honeycomb lattice. Finally, we predict the degenerated modes at Γ and K points that can be realized for particular values of bending and torsional rigidities. For example, when the bending/torsional rigidities have special values, the Dirac-like cones, triple degenerated points and even the double Dirac cones can be obtained. As reported in many previous publications, manipulation of Dirac cones could lead to many interesting effects [35–39]. For instance, by breaking the symmetry of the system, the opening of the gap in the Dirac K point in acoustic/elastic systems can give rise to the topological edge states propagating only along some particular directions [40–42]. Even for the Dirac cone at Γ point, one could expect to observe the pseudospin-resolved Berry curvatures of photonic bands and helical edge states characterized by Poynting vectors [43,44]. The analytical predictions of Dirac cones and double Dirac cones in this work could largely facilitate the study of the topological properties of elastic waves in more complex granular membranes with modified/broken symmetries. The study of these types of membranes is motivated by potential applications of the unidirectional waves propagating with reduced/avoided attenuation/scattering. In general, we believe that our theoretical analysis of the elastic waves in mechanically free membranes would be useful also in the studies of the interaction of the granular layers with the elastic substrates [45–48].

This paper is constructed as follows: in Section 2, the structures of the studied membranes and the interactions between beads are analyzed. The theoretical calculation and analysis of the modes in hexagonal monolayer membranes with out-of-plane motion is presented in Section 3. This analysis includes the phonon spectra, the zero-frequency modes, the degenerated modes and the Dirac cones at the high symmetry points. Then we are focusing on in-plane motion in hexagonal monolayer membrane in Section 4. In Sections 5 and 6, we turn our attention to the honeycomb monolayer membranes for both the case of out-of-plane and in-plane motions. In Section 7, we present the conclusions of this work.

2. Structures and intergrain interactions

As shown in Fig. 1, the two-dimensional infinite monolayer membranes under consideration are composed of periodically ordered spherical particles with radius R, arranged in a hexagonal and a honeycomb lattice. The structures are characterized by the lattice constant $a = 2R$ for the hexagonal lattice and $a = 2\sqrt{3}R$ for the honeycomb lattice. The corresponding first Brillouin zones are also depicted in Fig. 1. Considering different types of motion in these monolayer membranes, the following degrees of freedom are taken into account: (1) For the out-of-plane motion, the beads in the membrane exhibit out-of-plane displacement (u) along z-axis and in-plane rotational angles φ and ϕ (φ-rotation with the axis in the x-direction and ϕ-rotation with the axis in the y-direction). (2) For the in-plane motion, the beads in the membrane possess out-of-plane rotation (φ) along z-axis and in-plane displacements u_x and u_y along x-axis and y-axis, respectively. The dynamics and the coupling of these mechanical motions are controlled by the following forces and/or moments (see Fig. 2): (1) Shear forces, which are characterized by an effective shear rigidity ξ_s (Fig. 2(a)). These forces are activated in the membrane due to a resistance of the contact to relative displacement of the beads in the direction transversal to the axis connecting their centers and due to in-phase rotation of the beads relative to the direction normal to the axis connecting their centers. (2) Torsional forces, which are characterized by an effective torsional (spin) rigidity ξ_t (Fig. 2(b)). The resistance of the contact to relative rotation of the beads along the axis connecting their centers can initiate these forces. (3) Bending forces, which are characterized by an effective bending rigidity ξ_b (Fig. 2(c)). They originate from the resistance of the contact of beads to rolling. (4) Normal forces at the contact between two adjacent particles described by normal rigidity ξ_n (Fig. 2(d)). This type of interaction can be excited when there is relative displacement between two adjacent beads along the axis connecting their centers. For the out-of-plane motion, the motions of beads in the membranes lead to the shear, torsional and bending interactions, while the normal forces are not initiated. For the in-plane motion, the normal, shear and bending interactions are activated, while the torsional interactions are not.

In Fig. 2(a) the effective shear rigidity of the contact is represented by a single spring of an effective rigidity ξ_s and the energy of interaction can be evaluated because the relative macroscopic displacements of the neighbor beads are known. The interaction energy is proportional to the product of the effective shear rigidity and the square of the relative displacement which is equal to the elongation of the effective spring. However, one should keep in mind that the shear interactions are in fact distributed at the complete surface of the contact. To appreciate the role of the finite dimensions of the contact, we split the effective shear spring into two independent springs of half rigidity separated from spatially not only themselves but also from the center of the contact. The characteristic distance of the separation is of the order of the contact radius δ. This presentation of the shear interaction between the contact faces does not change the magnitude of shear interaction, however, it reveals the existence of the torsional rigidity of the contact. From Fig. 2(b) it is clear that, when the neighbor beads exhibit unequal rotations relative to the axis connecting their centers, one of the springs elongates while the other shrinks. Thus, it is the induced forces acting on the beads that are totally compensated being combined destructively, but not the moments. The moments due to the deformation of two beads will be added constructively, introducing resistance to torsional motion, which we describe as torsional interaction (Fig. 2(b)). It is worth noting that the elastic energy stored in shear interaction, presented in Fig. 2(a) is $\sim \xi_s(u_2 - u_1)^2$. The elastic energy stored in shear

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