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Short Communication

Rate of shear of an ultrasonic oscillating rod viscosity probe

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ABSTRACT

Ultrasonic oscillating rod probes have recently been used by researchers to measure viscosity and/or density in fluids. However, in order to use such probes to characterise the rheological properties of fluids, it is necessary to define the shear rate produced by the probe. This paper proposes an analytical solution to estimate the shear rate of ultrasonic oscillating rod viscosity probes and a method to measure their maximum operational shear rate. A relationship is developed which relates the torsional surface velocity of an oscillating cylindrical rigid body to the rate of shear in its vicinity. The surface displacement and torsional surface velocity of a torsional probe of length 1000 mm and diameter 1 mm were measured over the frequency range from 525 to 700 kHz using a laser interferometer and the maximum shear rate estimated. The reported work provides the basis for characterising shear rate for such probes, enabling their application for rheological investigations.

1. Introduction

The rheological characteristics of fluids influence many aspects of their performance, such as pumpability, droplet breakup in spray drying, emulsion formation, flow into moulds, formability and so forth, and qualities such as stability during and after processing. Hence, quantitative knowledge of the rheological properties of liquids and slurries is a crucial requirement for process design, process control, optimisation and the production of consistent quality products in many process environments such as petro- and speciality-chemicals, oil/gas, food, pharmaceutical, pulp, paper and nuclear. Among the rheological properties of fluids, viscosity is an integral component of a large number of quality control procedures in the processing of complex liquids [1].

Monitoring viscosity on-line and in-situ provides real-time data that can be used to optimise the process and support product quality. Viscosity can be expressed as the proportionality of the force (the shear stress) to the relative rate of movement (the rate of shear strain or shear rate). This proportionality can be independent of shear rate, in the case of Newtonian fluids, or vary with shear rate for non-Newtonian fluids. It is common to present viscosity as a function of shear rate in conventional methods of viscosity measurement. Hence, knowledge of the *shear rate* is of paramount importance in the study of fluids and their rheological characteristics such as viscosity.

Ultrasonic oscillating rod viscosity probes have been used to measure viscosity in liquids by many researchers [2–10]. This type of viscosity probe is robust and contains no moving parts and also does not represent hygiene risks. The instrument is of low cost so that it can be disposable after use. Therefore, there is a great interest for such an instrument in industrial applications where knowledge of its characteristics such as shearing rate can be beneficial.

In operation a rod, usually of metal, is immersed in the liquid to be tested. Transducers excite ultrasonic torsional (twisting) waves at one end of the rod, and these travel along the rod and are reflected back to the transducer either from the end of the rod or from an embedded discontinuity part way down. The echoes thus received are amplified and processed to extract the propagation velocity and/or the attenuation of the waves in the rod. Both velocity and attenuation are affected by coupling between the motions of the rod surface associated with the propagating wave in the rod and motions in the liquid in contact with the rod. Thus, depending on the wave mode, either the attenuation or the velocity can be used to calculate the viscosity of the surrounding liquid.

Viscosity measurements made with ultrasonic oscillating rod viscosity probes have been reported in a number of works. When testing general purpose Cannon hydrocarbon oil viscosity standards (Cannon Instrument Company, State College, PA, USA) with an ultrasonic oscillating rod operating in the 50–100 kHz range Kim and Bau [2], Costley et al. [5] and Vogt et al. [8] showed that the measurements agreed well with viscosity measurements using conventional rheometers. However, Ai and Lange [9] did not find good agreement in the higher frequency range 300–400 kHz for

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Nomenclature

$\mathbf{V}$	shear wave velocity field	ω	angular frequency
$\boldsymbol{\Phi}$	vector potential	η	dynamic viscosity
k	wave number	ρ	density
ν	kinematic viscosity	$\dot{\gamma}_{max}$	maximum shear rate

Cannon polybutene viscosity standards. Similarly, Shepard et al. [7] were unsuccessful in measuring general purpose Brookfield silicone fluid viscosity standards (Brookfield Engineering Laboratories Inc., Middleboro, Massachusetts, USA) in the frequency range 50–100 kHz and a varying shearing rate with an estimated effective shear rate about 25 s^{-1}; they related the inaccurate results to non-Newtonian behaviour of such liquids under relatively low and varying shear rate. Successful comparisons were reported by Shepard et al. [7] and Vogt et al. [8] for simpler liquids such as glycerol/water mixtures. An example of high temperature operation was demonstrated by Costley et al. [5] who measured the viscosity of molten glass; the shear rate was not defined. Recently, Rabani et al. [10] showed the oscillating rod viscosity probe to work well when applied to Newtonian fluids; however, anomalous behaviour was observed in the measurement of the viscosity of engine, gear and silicone oils.

The measured viscosities of these materials appeared to be very much lower than the values obtained with a conventional rheometer, using an oscillating rod with operating frequencies from 500 kHz to 700 kHz and an estimated maximum shear rate of 5000 s^{-1}. This anomalous behaviour was explained by molecular relaxation phenomena, which has a significant effect on the viscosity of polymer-based fluids at the probe frequency due to their long chain molecules. The data were successfully fitted to a relaxation model [10].

It would thus appear that the ultrasonic oscillating rod viscosity probe successfully measures the viscosity at its operating frequency, which may be different from the viscosity at a lower frequency. Since for non-Newtonian fluids, viscosity is strongly dependent on shear rate, it is important to characterise the shear rate produced by the probe. This paper comprises an analytical solution for the estimation of apparent shear rate in a cylindrical rod under torsional oscillation surrounded by a viscous medium and an experimental programme to determine indirectly the share rate. Surface displacement and velocity of an oscillating rod viscosity probe were measured and the maximum shear rate was estimated from these data. As far as the current authors are aware, these important considerations have not been discussed in the literature to date.

2. Theoretical basis

Propagation of torsional stress waves along the rod results in energy leakage into the surrounding fluid in the form of bulk shear waves. The fluid adjacent to the rod can be considered incompressible since the compressional waves play no part and the fluid movement can be considered laminar with constant velocity on any cylindrical locus concentric to the rod [11].

Defining cylindrical polar coordinate system, (r, θ, z), for the rod with z axis along the rod axis (Fig. 1), the velocity field for shear waves, = (v_r, v_θ, v_z), produced in the fluid by the probe can be expressed using a vector potential $\boldsymbol{\Phi}$ such that:

$$\mathbf{V} = \nabla \times \boldsymbol{\Phi} \tag{1}$$

where $\boldsymbol{\Phi}$ satisfies the Helmholtz equation:

$$\nabla^2\boldsymbol{\Phi} + k^2\boldsymbol{\Phi} = 0 \tag{2}$$

with k wavenumber:

$$k = (1+i)\left(\frac{\omega}{2\nu}\right)^{\frac{1}{2}} \tag{3}$$

where ν is the kinematic viscosity of the fluid $\left(\nu = \frac{\eta}{\rho}\right)$ and ω is the angular frequency of the bulk shear waves.

Fig. 1 shows cylindrical polar coordinates at a cross-sectional plane of the probe with axial symmetry and the illustrated configuration. In this configuration the vector potential has only z component, hence, $\boldsymbol{\Phi} = (0, 0, \varphi)$ and the appropriate solution for this geometry, with harmonic dependence, for a single frequency, ω, is in the form of Hankel functions [12]. Since ka is large, the solution can be taken in the limit of $kr \gg 1$ with an amplitude of φ_0, so that:

$$\varphi(r,t) = \varphi_0 \frac{e^{i(kr-\omega t)}}{\sqrt{kr}} \tag{4}$$

From Eqs. (1) and (4), the velocity in the fluid is:

$$v_\theta(r,t) = -ik\varphi_0 \frac{e^{i(kr-\omega t)}}{\sqrt{kr}} \tag{5}$$

For the simple shear case, the maximum shear rate is:

$$|\dot{\gamma}_{max}| = \left(\frac{\partial v_\theta}{\partial r}\right)_{max} = \left(\frac{\omega}{\nu}\right)^{\frac{1}{2}} |v_{\theta_{max}}| \tag{6}$$

where $|v_{\theta_{max}}|$ is the velocity amplitude at $r = a$. Thus, by measuring $|v_{\theta_{max}}|$ one can estimate the amplitude of the shear rate in the fluid at the surface of an oscillating cylindrical rod.

3. Experiments

Measurement of surface displacement and corresponding surface velocity of the oscillating rod is required for the estimation of rate of shear induced by the rod oscillation. A series of experi-

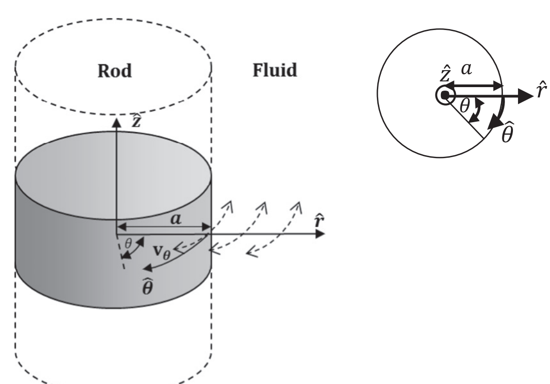

Fig. 1. Cylindrical polar coordinate system for the probe and velocity at the rod – fluid interface.