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Analytical and numerical modeling of non-collinear shear wave mixing at an imperfect interface

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ABSTRACT

Non-collinear shear wave mixing at an imperfect interface between two solids can be exploited for nonlinear ultrasonic assessment of bond quality. In this study we developed two analytical models for nonlinear imperfect interfaces. The first model uses a finite nonlinear interfacial stiffness representation of an imperfect interface of vanishing thickness, while the second model relies on a thin nonlinear interphase layer to represent an imperfect interface region. The second model is actually a derivative of the first model obtained by calculating the equivalent interfacial stiffness of a thin isotropic nonlinear interphase layer in the quasi-static approximation. The predictions of both analytical models were numerically verified by comparison to COMSOL finite element simulations. These models can accurately predict the additional nonlinearity caused by interface imperfections based on the strength of the reflected and transmitted mixed longitudinal waves produced by them under non-collinear shear wave interrogation.

1. Introduction

Non-collinear shear wave mixing has been reported to exhibit potential for assessing the additional nonlinearity caused by both bulk material degradation [1–4] and interface imperfections between solids [5,6]. When this technique is used to assess bulk nonlinearity, certain phase-matching resonance conditions must be satisfied so that nonlinear mixing of the two vertically polarized non-collinear shear waves can generate a third longitudinal wave with frequency and wave vector equal to the sum of the frequencies and wave vectors of the two interacting shear waves [7–10]

$$\omega_3 = \omega_1 + \omega_2, \tag{1}$$

$$\mathbf{k}_3 = \mathbf{k}_1 + \mathbf{k}_2, \tag{2}$$

where ω_i and $\mathbf{k}_i$ denote the angular frequencies and wave vectors, respectively, of the non-collinear shear waves ($i = 1,2$) and the mixed longitudinal wave ($i = 3$). The bulk resonance condition

$$\cos\varphi = c^2 - \frac{(1-c^2)(1+a^2)}{2a}, \tag{3}$$

determines the necessary angle φ subtended by the two interacting shear waves to assure that the periodicity of the interference pattern produced by them matches the wavelength of the longitudinal wave so that a cumulative mixing effect occurs over the whole interaction volume of the two shear waves. Here, $a = \omega_2/\omega_1$ denotes the frequency ratio and $c = c_s/c_d$ denotes the shear-to-longitudinal velocity ratio in the material.

When this technique is used to assess the nonlinearity of an imperfect plane interface between two half-spaces, the above resonance condition can be relaxed to suppress longitudinal wave generation by bulk nonlinearity in the surrounding material and thereby assure that the measured longitudinal wave is primarily caused by nonlinear mixing of the two shear waves at the interface to be inspected. This method was successfully used in the past to improve the sensitivity of the non-collinear shear wave mixing technique for interface imperfections in diffusion bonded T–6Al–4V specimens [5]. However, even when misaligned from the bulk resonance condition, this technique is still somewhat sensitive to the intrinsic bulk nonlinearity of the surrounding material, therefore the transmitted longitudinal wave will be proportional to a combination of material and interface nonlinearities. As a result, quantitative characterization of high-quality diffusion bonds was found difficult based on this technique alone [5]. In an effort to further reduce the adverse influence of bulk nonlinearity in the host material itself, in this study we conducted a combined analytical and computational investigation to identify the optimal inspection conditions for ultrasonic characterization of imperfect interfaces based on non-collinear shear wave mixing.

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Though the ultrasonic behavior of an imperfect interface between two solids is generally too complicated to accurately model, numerous investigations have been conducted in the past to capture the main features of both linear and nonlinear interaction of ultrasonic waves with typical imperfect interfaces. Most commonly used ultrasonic interface characterization schemes rely on linear reflection and, to a lesser degree, transmission measurements and exploit the characteristic frequency dependence of the scattering caused by interface imperfections [11–16]. Though widely used because of their simplicity, linear ultrasonic interface characterization techniques suffer from two main disadvantages. First, weak interface imperfections often remain hidden by coherent reflection and loss of transmission that occur even at perfectly bonded interfaces when two dissimilar materials are joined. Second, weak interface imperfections might remain undetected even in similar bonds, i.e., in the absence of impedance mismatch at the interface, because strong incoherent scattering from the microstructure of the host material can hide weak interface signals from poorly bonded but tightly closed interfaces that are often referred to as "kissing" bonds.

It has been recognized for many years that the nonlinear acoustic response of an imperfect interface is likely to change more significantly than its linear signature as the bond quality of the interface starts to drop. Some of the interface characterization techniques that exploit this fact are quasi-nonlinear in the sense that they use what is essentially linear ultrasonic inspection and measure the modulation that appears in the linear response of the interface under static or very low-frequency dynamic loading of the bond [17–21]. In contrast, truly nonlinear inspection methods rely on high-frequency self-modulation, i.e., harmonics generation by a large-amplitude ultrasonic wave, or cross-modulation, i.e., mixing between two waves at least one of which produces high-amplitude vibration [22–33]. In principle, there is no fundamental difference between these quasi-linear and nonlinear inspection methods. They all exploit the breakdown of the law of superposition in the presence of nonlinearity and together they cover a wide range of applications in ultrasonic interface characterization.

In a recent paper, An et al. developed a nonlinear spring model and reported that this model is capable of predicting second harmonic generation by obliquely incident longitudinal and horizontally polarized shear (SH) waves at an imperfect interface [34]. The present study is aimed at applying similar methods based on the nonlinear interfacial stiffness and thin nonlinear interphase layer models to predict the strength of non-collinear shear wave mixing at an imperfect plane interface that is normal to the wave vector of the mixed longitudinal wave therefore also to the plane of incidence determined by the wave vectors of the two interacting shear waves. In a recent paper Demcenko et al. studied a related phenomenon, namely non-collinear mixing of a shear wave with a longitudinal wave at an imperfect interface [35]. However, in the cases considered in [35] the imperfect interface was parallel rather than normal to the mixed longitudinal wave and the authors found that under those conditions the interface acts more as a barrier to nonlinear mixing than as the source of it. In Section 2 we will present our analytical results based on these two interface models that indicate that in the case of shear wave mixing at an imperfect interface the additional nonlinearity produces both reflected and transmitted longitudinal waves. Such symmetric generation of mixed nonlinear longitudinal waves was recently observed by Blanloeuil et al. using finite element simulations of closed cracks exhibiting contact acoustic nonlinearity [36]. Since the longitudinal wave generated by bulk nonlinearity always propagates in the transmission direction, detecting the nonlinear reflection from the imperfect interface could, in principle, eliminate the influence of the bulk nonlinearity and thus increase the sensitivity of the inspection to bond imperfections. In Section 3 these analytical predictions will be numerically verified by two-dimensional finite element (FE) simulations. Finally, in Section 4 we will draw some conclusions from the obtained results. For the interest of conciseness, additional algebraic details of the derivations are presented separately in the following Appendices A and B.

2. Theory

Fig. 1 shows a schematic illustration of non-collinear shear wave mixing at an imperfect plane interface. To simplify the problem, the media on the two sides of the interface are assumed to be identical. It should be pointed out that nonlinear ultrasonic inspection techniques are inherently far more complex than their conventional linear counterparts, therefore in practice nonlinear techniques are called upon only when simpler linear methods fail. Therefore, we will focus on interface imperfections that remain hidden from linear ultrasonic inspection because they do not produce measurable increase in reflection and decrease in transmission at the highest feasible inspection frequencies. Accordingly, Fig. 1 shows only the two obliquely incident and perfectly transmitted shear waves and the longitudinal reflected and transmitted waves produced by nonlinear mixing, but no reflected shear waves.

In order to assure that the wave vector of the nonlinear longitudinal wave is normal to the interface, the incident angles of the two primary shear waves are chosen according to Eq. (2), i.e., the tangential wave vector components of the two shear waves cancel each other

$$\omega_1 \sin\theta_1 = \omega_2 \sin\theta_2. \tag{4}$$

However, the periodicity condition for $\varphi = \theta_1 + \theta_2$ previously given in Eq. (3) do not have to be enforced. Actually, relaxing the periodicity condition can reduce longitudinal wave generation in the surrounding nonlinear host medium without influencing the sought contribution from the imperfect interface. Therefore, such misalignment can be used to suppress bulk nonlinearity in nonlinear interface characterization based on the non-collinear shear wave mixing technique [5].

The analytical method to be presented in this paper is based on the assumptions that (i) two plane waves interact at an infinite plane interface, (ii) the imperfect interface does not interfere with the incident waves, i.e., cannot be detected by linear ultrasonic inspection, and (iii) the host material is perfectly linear, i.e., the interface is the sole source of nonlinear mixing. This method, though not the explicit results derived in this paper, could be

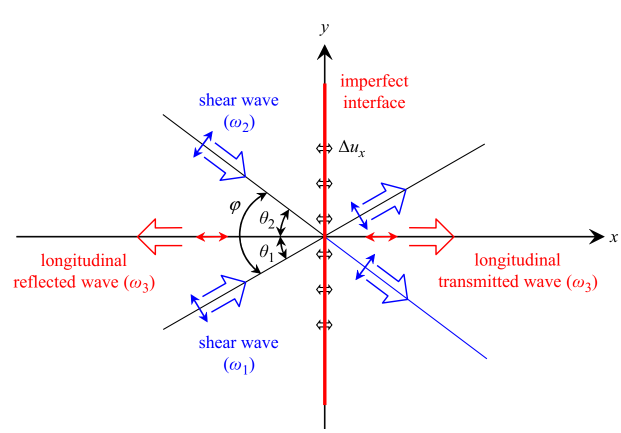

Fig. 1. Coordinate system used to study shear wave mixing at an imperfect interface. Only the two obliquely incident shear waves and the longitudinal reflected and transmitted waves produced by nonlinear mixing are shown.