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Propagation of ultrasonic Love waves in nonhomogeneous elastic functionally graded materials

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ABSTRACT

This paper presents a theoretical study of the propagation behavior of ultrasonic Love waves in nonhomogeneous functionally graded elastic materials, which is a vital problem in the mechanics of solids. The elastic properties (shear modulus) of a semi-infinite elastic half-space vary monotonically with the depth (distance from the surface of the material). The Direct Sturm–Liouville Problem that describes the propagation of Love waves in nonhomogeneous elastic functionally graded materials is formulated and solved by using two methods: i.e., (1) Finite Difference Method, and (2) Haskell-Thompson Transfer Matrix Method.

The dispersion curves of phase and group velocity of surface Love waves in inhomogeneous elastic graded materials are evaluated. The integral formula for the group velocity of Love waves in nonhomogeneous elastic graded materials has been established. The effect of elastic non-homogeneities on the dispersion curves of Love waves is discussed. Two Love wave waveguide structures are analyzed: (1) a nonhomogeneous elastic surface layer deposited on a homogeneous elastic substrate, and (2) a semi-infinite nonhomogeneous elastic half-space. Obtained in this work, the phase and group velocity dispersion curves of Love waves propagating in the considered nonhomogeneous elastic waveguides have not previously been reported in the scientific literature. The results of this paper may give a deeper insight into the nature of Love waves propagation in elastic nonhomogeneous functionally graded materials, and can provide theoretical guidance for the design and optimization of Love wave based devices.

1. Introduction

Shear horizontal (SH) surface Love waves have long been used in many fields of science and technology, e.g., in geophysics [1], seismology [2], non-destructive testing (NDT) of materials [3,4] and for determining the physical properties of materials. Sensors based on Love waves (due to their high sensitivity) are used for measuring physical properties of the liquid (e.g., viscosity and density) [5–7] as biosensors [8] and chemo-sensors [9], to investigate of thin films [10] and layers produced in the surface region of the substrate as a result of various technological processes (diffusion, implantation, carburizing, nitriding, shot peening, the laser treatment, etc.) [11], and also for testing of composites [12]. The use of layered Love waves waveguides with a nonhomogeneous distribution of physical properties can significantly improve performance (e.g., sensitivity and selectivity) of bio and chemosensors that employ the inhomogeneous elastic waveguides [13].

SH surface acoustic waves (Love and Bleustein–Gulyaev type) may also be used to study spatial profiles changes in mechanical properties (e.g., modulus of elasticity and density) of the Functionally Graded Material (FGM) [14–17]. These materials are heterogeneous media, in which the mechanical parameters are functions of the distance from the surface into the bulk of the material. Functionally graded materials can provide elevated mechanical properties (e.g., high strength and hardness) and superior exploitation characteristics (e.g., crack, wear and corrosion resistance). The FGM are widely used in modern industry (e.g., automotive, aviation, aerospace and electronic) [18]. Love wave penetration depth depends on the frequency. Thus, by changing the frequency of the wave one can probe subsurface profiles of materials. Love wave energy is concentrated near the surface of the waveguide. For this reason, any disturbance in the material parameters in the surface region have considerable impact on the dispersion characteristics of the Love wave (i.e., velocity and attenuation). Therefore, the Love waves are particularly convenient to study the physical properties of inhomogeneous graded materials.

The aim of this study was to develop a theoretical model of the propagation of SH (Shear Horizontal) surface Love waves in

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functionally graded materials with a monotonic variation of the elastic properties with the depth (distance from the treated surface of the material).

Determination of the phase velocity dispersion curves and the distribution of the mechanical displacement (into the bulk of the material) of the Love wave, for known profiles of elastic parameters of the medium in which the Love wave propagates, constitutes a Direct Sturm–Liouville Problem. In this study, the following profiles of the elastic coefficient $c_{44}(x)$ were considered: (1) the square root profile $n = 1/2$, (2) linear profile $n = 1$, (3) quadratic profile $n = 2$, (4) power type profile $n = 10$, (5) step profile $n = \infty$, (6) exponential profile, (7), profile of the $1/\cosh^2$ type (similar to the Gaussian profile). Governing equations of motion and the appropriate boundary conditions are given. The Direct Sturm–Liouville Problem has been solved using the Finite Difference Method and Transfer Matrix Method (Haskell-Thompson) [19,20]. The article includes a comparison of Love wave dispersion curves derived by these two numerical methods. Moreover, the integral formula for the group velocity of Love waves propagating in elastic graded materials was established, which is a novelty.

The problem of the Love wave propagation in nonhomogeneous elastic graded media was previously analyzed using various approximate methods such as the method of Frobenius [21], the method of Peano [22] and the WKB method [23]. However, these methods are pre-computer era methods. Their use does not introduce significant advantages in relations with modern numerical methods such as Finite Difference Method (FDM), Finite Element Method (FEM) or the Transfer Matrix Method (TMM). TMM method was used to analyze the Love wave propagation in nonhomogeneous medium [24]. However, this study presents only a general description of the TMM method without giving specific examples of physically realistic nonhomogeneous profiles and the corresponding dispersion curves.

In this work the problem of Love wave propagation in nonhomogeneous graded media was solved using the TMM and FDM methods. Phase and group velocity dispersion curves of Love waves in selected nonhomogeneous elastic graded media (power-law type profiles, exponential profile and the profile of the type $1/\cosh 2$) were evaluated. According to the authors' best knowledge, evaluation of the phase and group velocity dispersion curves for these selected profiles of the elastic modulus $c_{44}(x)$ is a novelty.

The results obtained in this work can constitute the basis of the inverse procedure (Inverse Sturm–Liouville Problem) to determine profiles (as a function of depth) of the mechanical properties of inhomogeneous FGM resulting from the application of various technological processes of surface treatment. The results of this study also provide a more complete description (than those published in the scientific literature) of the propagation of Love waves in graded materials with various profiles of changes in elastic properties, e.g., in layered inhomogeneous microstructures used in MEMS (Micro Electro Mechanical Systems) and other microelectronic devices, in photonics and in acoustoelectronics [3,25].

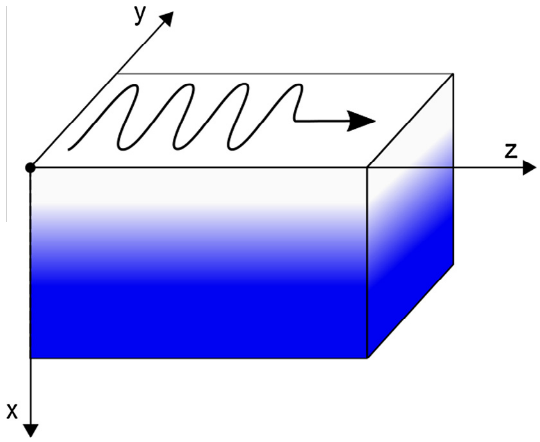

Fig. 1. Geometry of the Love wave waveguide structure (inhomogeneous elastic half-space), and coordinate system.

The results of this study can also find application in geophysics, earthquake engineering [26] and seismology to investigate the internal structure of Earth. Moreover, they can be very helpful in exploration of natural resources (e.g., gas and petroleum) [27].

Due to the similarity of the mathematical description of the phenomenon of propagation of Love waves in elastic inhomogeneous media and a description of the propagation of light waves in inhomogeneous planar optical waveguides, established in this work the theory of Love waves in elastic inhomogeneous media can also be used to analyze performance of inhomogeneous optical planar waveguides [28].

The results obtained in this paper are novel and fundamental and can give more profound insight into the nature of Love wave propagation in the elastic nonhomogeneous media (e.g., in functionally graded materials and composites).

Section 2 presents a mathematical model of the propagation of Love waves in the graded materials formulated as a Direct Sturm–Liouville Problem. Section 3 shows the considered shear modulus profiles $c_{44}(x)$ in the graded medium. Description of numerical methods applied to solve the Direct Sturm–Liouville Problem (i.e., the Finite Difference Method and Transfer Matrix Method) is included in Section 4. Section 5 contains the results of numerical calculations and discussion of the results. Finally, conclusions are presented in Section 6.

2. Direct Sturm–Liouville Problem

Love wave propagation in inhomogeneous elastic media can be described in terms of the Sturm–Liouville Direct Problem. Determination of the phase velocity and mechanical displacement distribution with depth of the SH surface Love wave from a knowledge of elastic parameters of a non-homogeneous half-space constitutes a Direct Sturm–Liouville Problem.

2.1. Love waves

2.1.1. Formulation of the problem

Consider the Love wave that propagates in a nonhomogeneous elastic half-space, as shown in Fig. 1. The elastic properties of inhomogeneous half-space vary monotonically with depth (distance from the surface).

Mechanical vibrations of the SH surface Love wave are performed along the y axis perpendicularly to the direction of propagation z and parallel to the propagation surface. The x axis is normal to the waveguide surface.

Mathematical description of the propagation of surface shear Love waves in graded media involves the use of continuum mechanics formalism to describe the motion of inhomogeneous elastic half-space.

SH surface wave of the Love type which propagates in a nonhomogeneous waveguide structure of Fig. 1 may be represented in the following form: $v(x,z,t) = V(x) \cdot \exp j(\beta z - \omega t)$, where: $V(x)$ is the distribution of the mechanical displacement of the Love wave with the depth, β is the wave propagation constant, $j = (-1)^{1/2}$, x is the distance from the surface (depth), z is the direction of wave propagation and ω is angular frequency.