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Investigation of complete bandgaps in a piezoelectric slab covered with periodically structured coatings

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The propagation of elastic waves in a piezoelectric slab covered with periodically structured coatings or the so-called stubbed phononic crystal slab is investigated. Four different models are selected and the effects of distribution forms and geometrical parameters of the structured coatings on complete bandgaps are discussed. The phononic crystal slab with symmetric coatings can generate wider complete bandgaps while that with asymmetric coatings is favorable for the generation of multi-bandgaps. The complete bandgaps, which are induced by locally resonant effects, change significantly as the geometry of the coatings changes. Moreover, the piezoelectric effects benefit the opening of the complete bandgaps.

1. Introduction

The study of elastic waves is always a research focus owing to its abundant potential applications in modern technology. When elastic waves propagate in periodic structures [1], such as phononic crystals (PNCs), some of them in a certain frequency range may be totally reflected and cannot pass through these periodic structures. This frequency range is termed bandgap. Recently, the propagation of elastic waves in stubbed PNC slabs has received increasing attention for their potential applications in acoustic devices [2–13]. Wu et al. [2] and Pennec et al. [3] independently demonstrated the existence of complete bandgaps and resonances in a slab with a periodic stubbed surface. And then, waveguiding of Lamb modes in the stubbed PNC slab structures were studied in [4,5]. The phenomenon of PNC-based filters, resonators and acoustic channels in the stubbed PNC slabs is discussed by Huang [7]. Other stubbed PNC slabs, such as two-layered stubs [8], stepped stubs [9], spiral resonators [10] and three-layered spherical structures [11] periodically deposited on the slabs, were also investigated. Assouar et al. [12] and Wang et al. [13] studied the bandgap and local resonance properties in double-side stubbed PNC slabs. In addition, Hassouani et al. theoretically demonstrated simultaneous existence of phononic and photonic bandgaps in a stubbed phoxonic crystal (PXC) slab [14].

Piezoelectric materials, as a kind of functional materials, have extensive applications in the fields of industry, biological medicine and national defense due to piezoelectric effects. Examples can be cited as sensors [15], sonars [16] and ultrasonic imaging [17], etc. Recently, piezoelectric materials are utilized in the periodic structures to form piezoelectric PNCs [18–32], which is popular in the design of new acoustic devices. Complete bandgaps have been obtained in a piezoelectric PNC slab by Khelif et al. [20] in 2006. Hsu and Wu [24] obtained locally resonant bandgaps for low-frequency Lamb waves in a piezoelectric PNC slab, and concluded that the resonant frequencies of the flexure-dominated slab modes are significantly dependent on the radius of the circular rubber fillers and the slab thickness. Hsu [27] investigated the effects of electrical boundary conditions on bandgaps in piezoelectric PNC slabs, and discussed the possibility to control frequency gaps only through changing the electrical boundary conditions. Recently, the propagation of elasto-electromagnetic coupled shear Bloch waves in a quasi one-dimensional (1D) periodic piezoelectric waveguide is studied within the full system of the Maxwell's equations [31].

In this paper, we study the propagation of elastic waves in a piezoelectric slab covered with periodically structured coatings, which can be regard as a piezoelectric stubbed PNC slab. As we indicated, some research works were devoted to this kind of PNC slabs. However, for piezoelectric PNCs, few papers have reported to study the propagation characteristics of Lamb waves. Here, we use the finite element method (FEM) to calculate the dispersion relation and transmission responses of Lamb waves propagating in this piezoelectric slab covered with periodically structured coatings. And the influences of distribution form and geometrical parameters of structured coatings, as well as the piezoelectric effects of the slab on the complete bandgaps are discussed. This

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paper is organized as follows: Section 2 presents the model and computational methods; Section 3 is devoted to the calculated results and the effects of geometrical parameters, piezoelectricity and electrical boundary condition on the bandgaps, while conclusions are drawn in Section 4.

2. Model and formulation

In the present work, an infinite piezoelectric (PZT-5H) slab covered with rectangular silver coatings, i.e., a kind of stubbed PNC slabs, is investigated. All coatings are periodically arranged in a square lattice along the x and y directions, respectively. Four different models are investigated, and the unit cells of these models are displayed in Fig. 1. As Fig. 1(a) shows, model (I) is a symmetric structure in which the piezoelectric slab is covered with the same coatings on each side. The lattice constant is a; the coating width is b; and the thicknesses of the piezoelectric slab and coatings are d and h, respectively. Models (II) and (III) are extended from model (I) by staggering the centers of the upper and lower coatings with a distance l. The centers of the coatings on each side of the piezoelectric slab in model (II) are staggered along the x direction, see Fig. 1(b); while the coating centers in model (III) are moved along the diagonal direction, as displayed in Fig. 1(c). Model (IV), which is covered with the coatings on one side of the piezoelectric slab distinguishing from the above three models is also taken into account.

We suppose that the polling direction of piezoelectricity in the slab is along the z-direction, perpendicular to the x–y plane. In most studies of piezoelectric devices the electric effect is treated as quasi-static, and electrodynamics (i.e. electromagnetic waves) is neglected [33]. The governing equation of elastic waves propagating in a piezoelectric slab can be expressed as

$$\begin{aligned} \nabla \cdot (\mathbf{C}(\mathbf{r}) : \nabla \mathbf{u}(\mathbf{r}) + \mathbf{e}(\mathbf{r}) \cdot \nabla \varphi(\mathbf{r})) &= \rho(\mathbf{r})\omega^2 \mathbf{u}(\mathbf{r}) \\ \nabla \cdot (\mathbf{e}^T(\mathbf{r}) : \nabla \mathbf{u}(\mathbf{r}) - \varepsilon(\mathbf{r}) \cdot \nabla \varphi(\mathbf{r})) &= 0 \end{aligned} \tag{1}$$

where $\nabla = (\partial/\partial x,\ \partial/\partial y,\ \partial/\partial z)$; $\mathbf{r} = (x, y, z)$ denotes the position vector, $\mathbf{C}(\mathbf{r})$ the elastic tensor, $\mathbf{u}(\mathbf{r})$ the displacement vector, $\rho(\mathbf{r})$ the mass density, ω the angular frequency, $\mathbf{e}(\mathbf{r})$ the piezoelectric tensor, $\varepsilon(\mathbf{r})$ the permittivity tensor and $\varphi(\mathbf{r})$ the potential. According to the Bloch theorem, the displacement filed and the electric field should satisfy

$$\boldsymbol{\psi}(\mathbf{r}) = e^{i(\mathbf{k}\cdot\mathbf{r})}\boldsymbol{\psi}_k(\mathbf{r}), \tag{2}$$

where $\boldsymbol{\psi}(\mathbf{r})$ denotes the displacements (u_x, u_y and u_z) or the electric potential (φ); $\mathbf{k} = (k_x,\ k_y)$ represents the wave vector limited to the first Brillouin zone of the reciprocal lattice; and $\boldsymbol{\psi}_k(\mathbf{r})$ is a periodic function with the same periodicity as the crystal lattice. At the interfaces between the piezoelectric slab and the stubs the displacement is continuous, which is the default interface condition in the FEM simulations. The silver coatings are considered as a purely elastic metal material, and thus the electrical boundary condition of the piezoelectric slab at the interface is open-circuit (charge free). For the surfaces of the piezoelectric material two electrical boundaries could be applied, i.e. open-circuit (charge free) and short-circuit (ground). For the open-circuit (short-circuit) boundary the normal electrical displacement (electrical potential) is zero at the surfaces. In this paper the open-circuit boundary is chosen as default boundary condition of the PZT surfaces. And in Section 3.4 the effect of different electrical boundary conditions on the bandgaps is discussed.

In this paper, the FEM is utilized to calculate the dispersion relations of the elastic waves propagating in the above systems. According to the characteristics of the structure, a three-dimensional unit cell is chosen to calculate the band structures. Neglecting the external applied force, the discrete form of the eigenvalue equations in a unit cell can be expressed as

$$\begin{bmatrix} \mathbf{K}_{uu} - \omega^2 \mathbf{M}_{uu} & \mathbf{K}_{u\varphi} \\ \mathbf{K}_{\varphi u} & \mathbf{K}_{\varphi\varphi} \end{bmatrix} \begin{pmatrix} \mathbf{u} \\ \varphi \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \tag{3}$$

where $\mathbf{K}_{uu}$ and $\mathbf{M}_{uu}$ denote the pure elastic stiffness and mass matrices, respectively; $\mathbf{K}_{u\phi}$ and $\mathbf{K}_{\phi u}$ are the piezoelectric-coupling stiffness matrices; $\mathbf{K}_{\phi\phi}$ accounts for the pure dielectric material; $\mathbf{u}$ and φ represent the displacement matrices and potential at the nodes, respectively. According to the Bloch theorem in Eq. (2), the displacements and electric potential for the nodes on the periodic boundaries of the unit cell satisfy

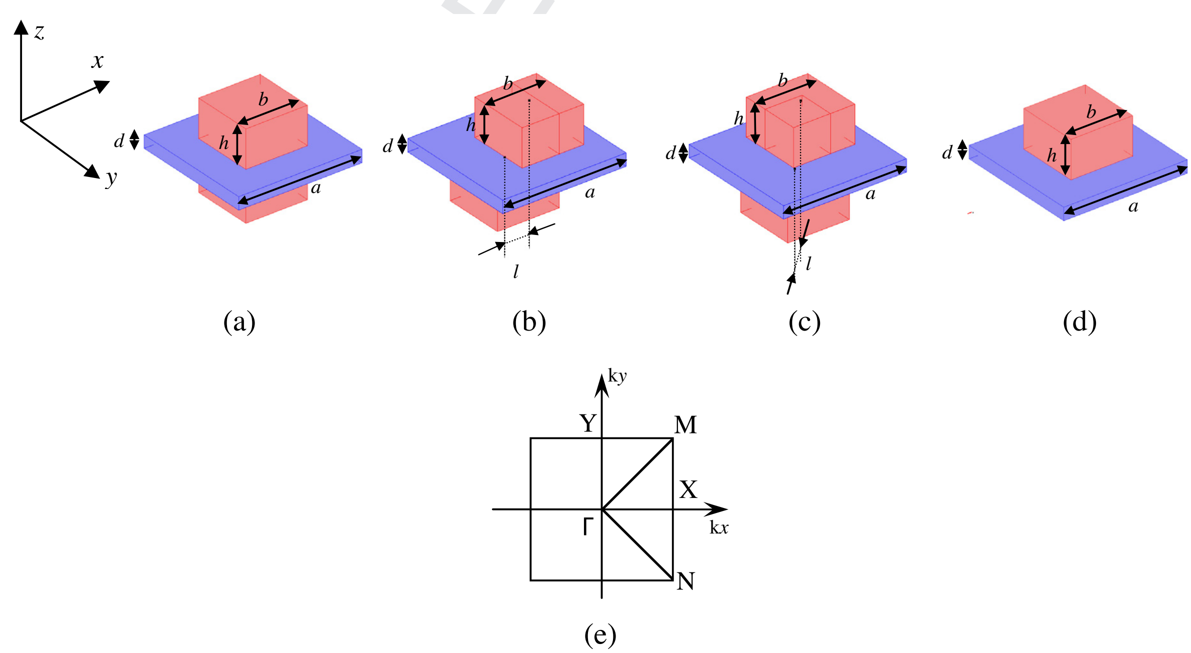

Fig. 1. (a)–(d) Unit cells of four different piezoelectric slab models covered with rectangular coatings. The blue and red regions are the PZT-5H slabs and the silver coatings, respectively. (e) Corresponding first Brillouin zones. The boundaries of the first Brillouin zones for models (I)–(IV) are Γ–X–M, Γ–X–M–Y, Γ–N–X–M and Γ–X–M, respectively. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

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