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Retrieval of the equivalent acoustic constitutive parameters of an inhomogeneous fluid-like object by nonlinear full waveform inversion

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ABSTRACT

This study addresses the problem of the acoustic characterization of an inhomogeneous object such as a soft-tissue organ containing a cyst or tumor whose size and/or composition evolve either negatively due to increased disease or positively due to increased response to treatment. The so-called 'corrupted' binary object, probed by a transient, acoustic plane wave, is a tube composed of a homogenous fluid-like (or assumed as such) mantle (medium 1: three acoustic constitutive parameters, one geometric parameter) surrounding a homogeneous fluid-like (or assumed as such) core (medium 2: three acoustic constitutive parameters, one geometric parameter), immersed in a spatially-infinite, homogeneous fluid (host medium 0: two acoustic parameters). The complete inversion of the diffracted acoustic field response of this object involves the retrieval of seven (six acoustic and one geometric) parameters, assuming we know beforehand the outer radius of the tube and acoustic parameters of the host. An alternative to this time-consuming, hazardous (due to the ill-posed nature of the) procedure, is to minimize the discrepancy, between the full waveform response of the binary object to a transient plane wave and the response of a homogeneous cylinder (medium characterized by three acoustic parameters, one geometric parameter) to the same transient plane wave, so as to retrieve the (three so-called equivalent) acoustic parameters of the homogeneous object. Thus, the first inverse problem is replaced by a second one (same assumptions concerning the outer radius of the objects, the host medium, the probe radiation and the sensing configuration as the first one) involving the retrieval of only three (instead of six) acoustic parameters. This procedure is potentially useful if the variation of at least one of the three equivalent parameters is sensitive to the variation of a key parameter of the inhomogeneous body (usually the characteristic dimension or the wavespeed of the core) and this variation can be expressed in a simple algebraic form (such as by a mixing formula). It is shown that this situation can arise if the average frequency of the acoustic probe radiation is sufficiently low. A sidelight of this investigation is the discovery that the equivalent constitutive parameters of the homogeneous cylinder are dispersive even when the component materials of the tube are not dispersive.

1. Introduction

Suppose we want to determine the wavespeed (or other physical descriptor) of an object from its response to an impressed acoustic (or other (e.g., wave-like)) interrogating field. This is a well-defined *inverse problem* for the retrieval of one or two scalar constant parameters (the real and/or imaginary parts of the wavespeed or other physical descriptors) from response data *provided the object is isotropic and homogeneous*. The issue is less clear when the object is inhomogeneous (although still isotropic).

Suppose, the object is binary, i.e., composed of two homogeneous materials, and that we do not have precise information as to the physical properties of the constituents (this might not be the case if the object is man-made, but is often the case for natural or biological objects). Then the inverse problem is that of the retrieval of the two to four scalar constant parameters (i.e., the real and imaginary parts of the wavespeeds; two more parameters if the density is to be retrieved too) of the two component materials, and perhaps their spatial distribution, from the dynamic response. This may turn out to be a much more difficult inverse problem (sometimes called qualitative tomography when only the geometric features are retrieved [60], and quantitative tomography [30,31,105,51,12,38] when both the geometric and physical features are retrieved) because it involves the distribution of one

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material with respect to the other (shell-like, raisin pudding-like, aggregate-like, etc.), which requires knowledge of a host of other parameters that are generally unknown, so that the dimension (i.e., the number of (real) parameters which must be retrieved if they are all considered to be unknowns) of the inverse problem can, in fact, be much greater than four.

The question is then whether it might be of use to consider the inhomogeneous object as being homogeneous (or simpler in some other sense) and retrieve the wavespeed (and/or density) in the same manner (i.e., by processing experimental data concerning the response of the object to an interrogating field) as for the homogeneous (or simpler) object. More precisely, the question is whether the so-obtained (perhaps abusively-termed) "equivalent wavespeed" (or other equivalent parameter) might furnish useful information:

- as in monitoring the growth of a tumor or response to treatment in an organ [8,21,32,37,41,49,62,65,71,76,83,84,96,104,112,43,40,38],
- concerning plaque in an artery [27,29,117],
- as to heterogeneities in the abdomen and thorax [34,61,101,115],
- as to the physical state of bone [35,56,75],
- as to the possibility of cancer in lymph nodes, the thyroid gland, etc. [64,83,118],
- as to pathologies of the heart [73,74,81]
- as to the physical state of trees [14,68],
- as to defects, or the physical state, of food [58,67,93],
- as to corrosion or occlusion in pipes [6,7,69,94,95,114],
- as to the thickness of a wall or layer, volume, size of an inclusion [5,9,11,22,23,27,57,69,72,80,89,97,95,60].

More generally, our inquiry concerns whether the notion of "equivalent parameter" is appropriate for determining the nature and/or extent of any type of heterogeneity (layer, coating, object (s) within another object, object within a medium, inhomogeneous object, etc.) [13,26,25,36,44,48,50,52,54,39,53,82,88,97,99,100,116,45,46,58,85,86,90,109,110,55,78,101,43,38,80].

Note that if: (a) the object appears to us to be homogeneous from its exterior appearance, (b) we have no visual access from the outside to the interior of the object, and (c) we are unable or unwilling to obtain this information in destructive (surgical) manner, (d) the object is too far away to be interrogated other than by remote sensing [28,42,50], then we usually have no choice other than to treat the object as being homogeneous. This is, in fact what is usually assumed in material characterization (e.g., of the permittivity or wavespeed) studies (based on capacitance comparison, transmission line techniques, dielectrometry [63] detection of perturbations (shift of frequency and modification of the quality factor) of a resonant cavity [115], reflectometry [3], refractometry [1,2,10,3], ellipsometry [103] and diffractometry [36,42]). Moreover these techniques usually require that the material under study be (or is assumed to be [42,50]) in the form of a specimen of simple shape (e.g., block, slab, film, sphere, bar, etc.) so as to simplify the resolution of the associated inverse problem.

It is important to stress that, in the present investigation, we take the material characterization point of view (see the last few references and the more recent metamaterial-related ones: [19,20,47,91,92,111,113]) in that we suppose that the *experimental response* data (or simulations thereof) is what is employed to determine the equivalent parameters (i.e., we are not doing homogenization or any other type of theoretical averaging of material properties, all the more so than we assume the key material properties to be unknown). This requires a parameter retrieval model (e.g., in reflectometry, the model relates the reflectivity to the index of refraction) and this model can be more or less appropriate (due to: a too-simple mathematical model, to computational simplifications or error, to uncertainty of the so-called *priors* (whose values are not retrieved, but rather assigned, during the inversion) that enter into the model, ...) to describe the response [16,17,19,61,66,75,78,82,87,86,106,108–110].

The issue of how appropriate is the parameter retrieval model, especially as concerns the incidence of uncertainty of the parameters [15] that enter into the model will not be addressed in this work; the stress will rather be on the sensitivity of the equivalent parameters to key parameters of the inhomogeneous object and the influence of the spectral properties of the transient probe radiation as well as the number and positions of the sensors.

Our study will focus on the example of a binary object of simple shape: an infinitely-long circular cylinder containing an infinitely-long circular cylindrical inclusion (the combination of these two components is termed "tube").

1.1. Description of the scattering configurations

The z axis of an $Oxyz$ cartesian reference system constitutes the axis of an infinitely-long (in the z-direction) circular tube, termed hereafter the 'corrupted object'. The outer, healthy portion of the tube, termed mantle, is occupied by a homogeneous (or considered as such), lossy, fluid-like medium, whereas the inner, corrupted portion (e.g., cyst or tumor) of the tube, termed core, is occupied by another homogeneous (or considered as such), lossy, fluid-like medium. The tube is immersed in a spatially-infinite region occupied by a homogeneous, non-lossy fluid wherein is assumed to propagate a transient acoustic plane wave (termed probe radiation) whose wavevector $\mathbf{k}^i$ lies in the $x-y$ (sagittal) plane (see Fig. 1). The effect of the probe radiation on the corrupted object is to produce a transient diffracted field termed response. Since the incident field and the object(s) are independent of z, the diffracted field is likewise independent of z, i.e., the problem(s) is (are) 2D, with z the ignorable coordinate.

In the direct-scattering problem, the task is to predict the diffracted field, given the probe radiation and the various constitutive and geometric parameters of the object(s) and the host. In a complete parameter-retrieval (i.e., inverse) problem, the diffracted field (equal to the total measured field minus the incident field (the latter is measured in the absence of the object)) constitutes the data, which by means of an inversion scheme, is analyzed to enable the retrieval of the constitutive and geometric parameters of the corrupted object.

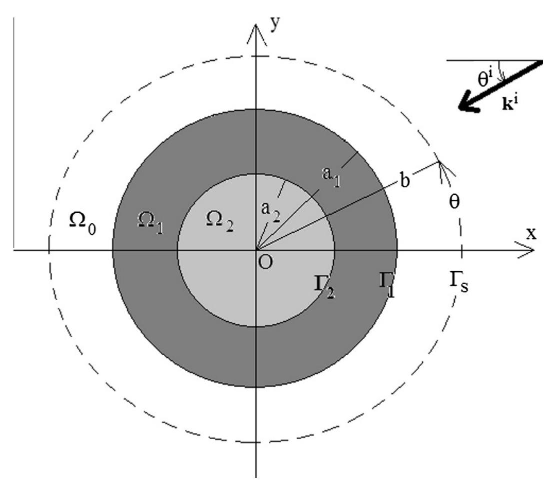

Fig. 1. Problem configuration for the tube-like corrupted object in the sagittal plane. The equivalent object, again of radius a_1, is formed by taking away the core ($a_2 \to 0$) and appropriate modification of the physical properties of the mantle.