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Research on bandgaps in two-dimensional phononic crystal with two resonators

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ABSTRACT

In this paper, the bandgap properties of a two-dimensional phononic crystal with the two resonators is studied and embedded in a homogenous matrix. The resonators are not connected with the matrix but linked with connectors directly. The dispersion relationship, transmission spectra, and displacement fields of the eigenmodes of this phononic crystal are studied with finite-element method. In contrast to the phononic crystals with one resonators and hollow structure, the proposed structures with two resonators can open bandgaps at lower frequencies. This is a very interesting and useful phenomenon. Results show that, the opening of the bandgaps is because of the local resonance and the scattering interaction between two resonators and matrix.

An equivalent spring-pendulum model can be developed in order to evaluate the frequencies of the bandgap edge. The study in this paper is beneficial to the design of opening and tuning bandgaps in phononic crystals and isolators in low-frequency range.

1. Introduction

Over the last two decades, much effort has been devoted to the study of the propagation of elastic waves in the periodic composite structures, the phononic crystals (PCs) [1–6]. PCs are artificial media composed of a periodical repetition of inclusions in a matrix with various topologies [7–10]. The great attention on these composites is mainly due to their rich physical properties such as negative refraction, localized defect modes, and phononic bandgaps (PBGs) [11–14]. Especially, the existence of the PBGs, in which sound and vibration are fully prohibited in any direction [15–17], makes the PCs being found in abundant potential applications in the design of acoustic devices such as acoustic filters, noise isolators, and acoustic mirrors [18–22].

To promote the application of PCs in noise control and mechanical engineering, the acquisition of tunable bandgaps with large bandwidth in low-frequency range is of great importance. For the two-dimensional PBG materials, a lot of researches have been carried out to explore PC structures with excellent bandgaps. Min et al. studied the effect of symmetry on the PBGs in two-dimensional binary phononic crystals including five types of straight rod arranged in hexagonal lattices [23]. Hu et al. presented a type of phononic crystal composed of an array of two-dimensional Helmholtz resonators and analyzed its properties of the relative acoustic refractive index and acoustic impedance mismatch of the structure [24]. Liu et al. designed a novel PC structure that exhibits PBGs two orders of magnitude smaller than the lattice constant and accordingly proposed the localized resonance (LR) mechanism [25]. Laude et al. identified the full bandgap for surface acoustic waves in a piezoelectric phononic crystal composed of a square-lattice Y-cut lithium niobate matrix with circular void inclusions [26]. Vasseur et al. demonstrated the existence of the guided modes inside a linear defect created by removing one row of holes in both a freestanding PZT5A slab and a slab deposited on a silicon substrate [27]. Hsu et al. reported the locally resonant bandgap of Lamb waves in binary PC slabs [28]. Pennec et al. studied the band structure of a PC slab constituted of a periodical array of cylindrical dots deposited on a thin slab of a homogeneous material [29]. Brunet et al. explored a silicon slab made of centered rectangular and square arrays of holes. No complete bandgap was found [30]. Mohammadi et al. fabricated a PC slab by etching a hexagonal array of air holes through a free standing slab of silicon. The measured high frequency bandgap matched very well with the theoretical simulations [22]. Sun et al. presented the theoretical and experimental investigation of the elasticwave propagation in PC slabs with membranes [31]. Straight and bent guiding are theoretically reported by Oudich et al. in a locally resonant PC

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structures composed of 2D silicon rubber stubs squarely deposited onto an epoxy finite plate [32]. Negative refraction experiments were conducted by Lee et al. with guided shear-horizontal waves in a thin PC plate with circular holes [33]. Wu et al. demonstrated the focusing of Lamb waves in a gradient-index PC slab [34]. The beam width of the lowest antisymmetric Lamb mode can be efficiently compressed. Zhu et al. investigated a thin metamaterial plate with different cantilever-mass systems. No complete bandgap is found in any system [35]. Kuo and Piazza examined the porous fractal PC slabs. Ultra high frequency bandgaps were found in both theory and experiment [36]. Cheng et al. studied the propagation of Lamb waves in PC slabs composed of periodic layers bilaterally deposited on both sides of the homogeneous core layer [37]. A theoretical study of Lamb wave propagation in a 2D PC slab composed of an array of solid Helmholtz resonators on a thin plate is reported by Hsu [38]. Wang et al. investigated the optimized design of alternate-hole-defect on a silicon PC slab in a square lattice. They found out that the Q factors are generally in an inverse relationship with the standard deviation of the band [39]. Leamy studied the dispersion relations of the honeycomb structures with square, diamond and hexagonal lattices and found out that only the hexagonal honeycomb exhibits a low frequency bandgap [40]. In a recent paper, Wang and Wang proposed a kind of PC slabs with cross-like holes [41]. Multiple wide complete bandgaps are found in a large range of thickness.

Up to now, research on the system with the resonators inside the plate has seldom been reported. Yu and Chen studied lamb waves in two-dimensional phononic crystal slabs with neck structure [42]. With the resonators depositing inside, the whole structure will be lighter as some material is removed, and meanwhile the volume is smaller. Therefore, the study on the propagation of Lamb waves in a PC plate with the resonators inside is more interesting. But, there is no report on the phononic crystal with two resonators.

In this paper, the two rectangular inclusions with neck structure are introduced into PCs as local resonators. The main effect is the possibility of opening a low-frequency bandgap for the slab waves, which may be applicable to the isolation of the low-frequency vibrations and manipulation of the low-frequency elastic wave with a smaller and lighter structure than a typical PC. Numerical simulation is implemented by use of finite element method. The effect of the connector layout and the influence of the geometry parameters on the band structure are investigated. The eigen modes at the bandgap edges are calculated to analyze the mechanism of the bandgap generation, and an optimal design of PC slab with two resonators which producing a large complete bandgap is presented as well.

2. Model and formulation

A kind of structure of PCs of an isotropic solid with two resonators in a square lattice is proposed. As shown in Fig. 1(a) and (b), the PC structure considered here is a slab with two rectangular inclusions embedded periodically along the X–Y plane. The inclusions are not connected with the slab directly but linked through some rectangular connectors. The structure is infinite in the Z-direction and the axes of the resonators are parallel to the surface of the PCs. The sidelength of the Model 1 is lattice constant a, and the four sides are equal in length. Inside the Model 1, the sidelength of hollow part is c. In the same cavity, there are two separate resonators, their lengths and widths are k_1, w_1, k_2, and w_2 respectively. The connector geomotry parameters are h_1, b_1, h_2, and b_2. The corresponding irreducible Brillouin zone of unit cell is shown in Fig. 1(c).

In the present work, to investigate the gap characteristics of these new kinds of PC structure, a series of calculations on the dispersion relationship and transmission spectra are conducted with FEA method based on the Bloch theorem [43–46]. For the calculation of the dispersion relations of the proposed structure referring to an infinite system, the governing field equations are given by

$$\sum_{j=1}^{3}\frac{\partial}{\partial x_j}\left(\sum_{l=1}^{3}\sum_{k=1}^{3}c_{ijkl}\frac{\partial^2 u_i}{\partial t^2}\right)=\rho\frac{\partial^2 u_i}{\partial t^2}\quad (i=1,2,3), \tag{1}$$

where ρ is the mass density, u_i is the displacement, t is the time, c_{ijkl} are the elastic constants, and x_j (j = 1, 2, 3) represents the coordinate variables x, y, and z respectively.

Since the infinite system is periodic along the x and y directions simultaneously, according to the Bloch theorem, the displacement field can be expressed as

$$u(r)=e^{i(k\cdot r)}u_k(r) \tag{2}$$

where $k = (k_x, k_y)$ is the wave vector limited to the first Brillouin zone of the repeated lattice and $u_k(r)$ is a periodical vector function with same periodicity as the crystal lattice.

In the present work, the finite element method (FEA) is used to calculate the structures of the PCs. a series of calculation on the dispersion relations and transmission spectra are conducted with the FEM. Due to the periodicity of PCs, the calculation can be implemented in a representative unit cell. The eigenvalue equations in the unit cell can be written as

$$(K-\omega^2 M)U=0 \tag{3}$$

where U is the displacement at the nodes and K and M are the stiffness and mass matrices of the unit cell, respectively. The Bloch theorem of Eq. (2) should be applied to the boundaries of the unit cell, yielding

$$U(r+a)=e^{i(k\cdot a)}U(r) \tag{4}$$

where r is located at the boundary nodes and a is the vector that generates the point lattice associated with the phononic crystals.

COMSOL Multiphysics 3.5a is utilized to directly solve the eigenvalue Eq. (3) under complex boundary condition of Eq. (4).

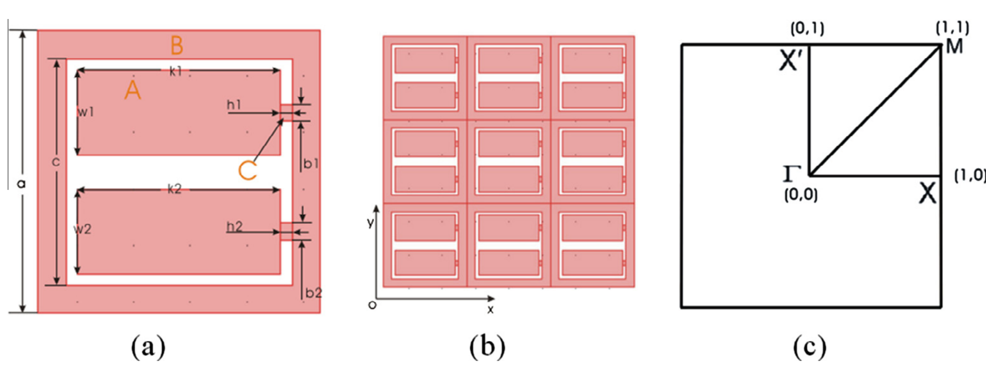

Fig. 1. Schematic view of the cross-sections of the PC structure Model 1 and the corresponding irreducible Brillouin zone.