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● *Original Contribution*

ANALYSIS OF TRANSIENT SHEAR WAVE IN LOSSY MEDIA

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Abstract—The propagation of shear waves from impulsive forces is an important topic in elastography. Observations of shear wave propagation can be obtained with numerous clinical imaging systems. Parameter estimations of the shear wave speed in tissues, and more generally the viscoelastic parameters of tissues, are based on some underlying models of shear wave propagation. The models typically include specific choices of the spatial and temporal shape of the impulsive force and the elastic or viscoelastic properties of the medium. In this work, we extend the analytical treatment of 2-D shear wave propagation in a biomaterial. The approach applies integral theorems relevant to the solution of the generalized Helmholtz equation, and does not depend on a specific rheological model of the tissue's viscoelastic properties. Estimators of attenuation and shear wave speed are derived from the analytical solutions, and these are applied to an elastic phantom, a viscoelastic phantom and *in vivo* liver using a clinical ultrasound scanner. In these samples, estimated shear wave group velocities ranged from 1.7 m/s in the liver to 2.5 m/s in the viscoelastic phantom, and these are lower-bounded by independent measurements of phase velocity. (E-mail: kevin.parker@rochester.edu)

Key Words: **Ultrasound, Elastography, Shear waves, Group velocity, Viscoelastic tissue.**

INTRODUCTION

A number of techniques have been developed to estimate and image the elastic properties of tissues (Doyley 2012; Parker et al. 2011). These provide useful biomechanical and clinically relevant information not available from conventional radiology. A subset of techniques utilize acoustic radiation force from short-duration pushing pulses as an initial condition, which then results in a propagating shear wave. Through tracking of the propagating wave, the shear wave velocity can be estimated, and this yields the Young's modulus—or stiffness—of the material (Sarvazyan et al. 1998). A variety of approaches employing radiation force, with important clinical applications, have been developed (Fatemi and Greenleaf 1998; Hah et al. 2012; Hazard et al. 2012; Konofagou and Hynynen 2003; McAleavey and Menon 2007; Nightingale et al. 1999; Parker et al. 2011).

In lossy tissues, however, a propagating shear wave produced by a focused ultrasound beam's radiation force will rapidly diminish within a few millimeters from the source. Furthermore, the displacement wave has an extended "tail," and its original shape becomes distorted. These effects complicate attempts to track the key features of the propagating pulse to estimate shear wave speed. Analytical and numerical models have been proposed to model the evolution and decay of pulses in viscoelastic media (Bercoff et al. 2004a; Fahey et al. 2005; Kazemirad et al. 2016; Leartprapun et al. 2017; Nenadic et al. 2017; Nightingale et al. 1999; Parker and Baddour 2014; Sarvazyan et al. 1998; Schmitt et al. 2010; Vappou et al. 2009; Wijesinghe et al. 2015). However, there is still the need for a closed-form analytical solution that clearly identifies the key terms responsible for the distortion and decay of the pulse. Furthermore, there are different models for wave propagation in lossy media (Bercoff et al. 2004b; Chen et al. 2004; Chen and Holm 2003; Giannoula and Cobbold 2008, 2009; Szabo 1994; Urban et al. 2009). Because there is no consensus yet as to the most appropriate model and mechanism of loss for shear waves in soft tissues, it is useful to have analytical expressions that are independent of any particular model, but still valid over the operating range of shear wave frequencies.

The approach taken in this article follows the earlier framework of Parker and Baddour (2014). First, the governing equations and transforms are stated in a progression

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favored by the classic treatment of Graff (1975). Then, a 2-D beam pattern is introduced, and the equations are reduced to simplified forms. General viscoelastic material properties are simplified to first-order (Taylor series expansion) terms and introduced into the analytic solutions, retaining leading terms. From these, some estimators of tissue parameters can be specified. Some preliminary examples are then presented, in which the data are taken from a clinical imaging scanner.

THEORY

We model the applied radiation force as being long and relatively constant in the z (depth) direction, so that spatial derivatives in the z direction are small compared with other terms. In practice, this is commensurate with a higher f-number focus in a weakly attenuating medium and multidepth push sequences. In this case, we assume that the following holds for displacements u and body forces f:

$$u_x = u_y = 0, \quad u_z = u_z(x, y, t)$$
$$f_x = f_y = 0, \quad f_z = f_z(x, y, t) \tag{1}$$

In these circumstances, the governing equations for displacements in the medium reduce to

$$\mu \nabla^2 u_z + \rho f_z = \rho \ddot{u}_z, \quad \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \tag{2}$$

where μ is the shear modulus and ρ is the density of the medium. The particle motions are polarized in a single direction z, and the resulting waves will be shear waves propagating at the velocity $c = \sqrt{\mu/\rho}$ (Graff 1975).

By taking the spatial and temporal Fourier transform of the governing equation, and then the inverse transform, we find the solution is given by

$$u_z(x, y, t) = \frac{1}{(2\pi)^{3/2}} \iiint \frac{F(\varepsilon, \eta, \omega)}{\varepsilon^2 + \eta^2 - (\omega^2/c^2)} \mathbf{e}^{i(\varepsilon x + \eta y - \omega t)} d\varepsilon d\eta d\omega \tag{3}$$

where $F(\varepsilon, \eta, \omega)$ is the Fourier transform of $c^2 f(x, y, t)$, the applied radiation force pulse. Assuming $f(x, y, t)$ is a sufficiently short pulse so as to be modeled as an impulse in time (Zvietcovich et al. 2017) and Gaussian in (x, y) with spatial width of (σ_x, σ_y), respectively,

$$F(\varepsilon, \eta, \omega) = 1\mathbf{e}^{-\frac{1}{2}\left(\sigma_x^2 \varepsilon^2 + \sigma_y^2 \eta^2\right)} \tag{4}$$

Substituting the particular form yields

$$u_z(x, y, t) = \frac{1}{(2\pi)^{3/2}} \iiint \frac{\mathbf{e}^{-\frac{1}{2}\left(\sigma_x^2 \varepsilon^2 + \sigma_y^2 \eta^2\right)}}{\varepsilon^2 + \eta^2 - (\omega^2/c^2)} \mathbf{e}^{i(\varepsilon x + \eta y - \omega t)} d\varepsilon d\eta d\omega \tag{5}$$

The direct solution of eqn (5) involves treatment of the singularity formed by the denominator becoming zero when $\varepsilon^2 + \eta^2 = \omega^2/c^2 = k^2$. Baddour (2011) has insightfully explained how the denominator serves as a "sifting" property, meaning the solution is completely governed by the integrand evaluated at the singularity. For example, Baddour's theorem 5 for complex exponentials and a real wave number is

$$I(k, r) = \int_{-\infty}^{\infty} \frac{\phi(\eta)}{\eta^2 - k_t^2} \mathbf{e}^{i\eta r} d\eta = \frac{i \cdot \pi}{k_t} \phi(k_t) \mathbf{e}^{ik_t r} \quad r > 0 \tag{6}$$

Effectively, this transforms the spatial transform $\phi(\eta)$ related to the distribution of force and converts it to a temporal transform $\phi(k)$, where the singularity caused by the denominator selects the value of k. Thus, considering the integration of eqn (5) over the spatial frequencies, we examine the quantity

$$\phi(k) = \iint \left(\mathbf{e}^{-\frac{1}{2}\left(\sigma_x^2 \varepsilon^2 + \sigma_y^2 \eta^2\right)} \right) \left(\mathbf{e}^{i(\varepsilon x + \eta y)} \right) d\varepsilon dn \tag{7}$$

on the circle defined by $\varepsilon^2 + \eta^2 = k^2$. Substituting $\varepsilon = k \cos \theta$, $\eta = k \sin \theta$, $d\varepsilon dn = r dr d\theta = |k| dr d\theta$, considering first the integration over r, and comparing with eqn (6) from Baddour's theorem 5, we have

$$u_z(x, y, k) = \frac{-i \operatorname{sign}(k)}{8\pi} \int_0^{2\pi} \mathbf{e}^{-\frac{k^2}{2}\left(\sigma_x^2 (\cos\theta)^2 + \sigma_y^2 (\sin\theta)^2\right)} \mathbf{e}^{ik(x\cos\theta + y\sin\theta)} d\theta \tag{8}$$

Rewriting the $\mathbf{e}^{-\frac{k^2}{2}(\ )}$ term for the case where $\sigma_y > \sigma_x$ (as is common in 1-D linear arrays, where y represents the elevational direction),

$$= \mathbf{e}^{-\frac{k^2 \sigma_x^2}{2}\left(\cos^2\theta + R^2 \sin^2\theta\right)} \tag{9}$$

where $R^2 = \sigma_y^2 / \sigma_x^2$, and could be 4 to 100 depending on the particular array.

No closed-form analytical solution to eqn (8) has been found. However, for the special case of radial symmetry, where $R = 1$, and on the x-axis, where $y = 0$, eqn (8) reduces to