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Spectral windows for satellite radar altimeters

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Abstract

A satellite altimeter's waveform is a power spectral density (PSD) estimate that displays backscattered power as a function of range, and in the Delay/Doppler or multi-looked SAR (D/D-SAR) algorithm, also of along-track position. Earth surfaces reflect radar power at a continuum of ranges and along-track positions, and so waveforms inevitably suffer from spectral leakage. Leakage may be mitigated, and the PSD resolution ("point target response", PTR) shaped, by employing a spectral window, in either or both of the range and along-track dimensions. This paper demonstrates the sampling, symmetry and zero-padding required to ensure that the window does not introduce any distortion of the waveform. Because the PTR shapes the waveform through convolution, this paper characterizes waveform resolution in terms of PTR integrals. Expressing these as matrix-vector quadratic forms shows that ideal windows may be built from eigenvectors of appropriate matrices. A new approach taken here is to seek windows that make the PTR as nearly Gaussian as possible, since a Gaussian PTR is assumed in theoretical models for the statistical expectation of waveforms from uniformly rough surfaces. Up to now, altimeters have used either rectangular or Hamming windows, but this paper proposes other windows that provide a narrower and more Gaussian PTR with adequate leakage suppression. CryoSat-2 SAR mode data over transponders and leads in sea ice are processed with various windows to demonstrate applications and results.

Keywords: Leakage; Optimization; Radar; Spectrum; Waveform; Window

1. Introduction

In satellite altimetry, the elevation, roughness and backscatter of Earth surfaces are estimated by fitting models to the altimeter's waveform, which is a power spectral density (PSD) estimate obtained from discrete Fourier transforms (DFTs) applied to time series produced by the instrument's analog-to-digital converter (ADC). In all altimeters the PSD estimation process is at least one-dimensional, and relates echo power to range. The "delay/Doppler" (Raney, 1998) or "multi-looked SAR" (Wingham et al., 2006) (hereafter, "D/D-SAR") algorithm adds a second dimension that exploits inter-pulse coherency to map echo power to along-track position. In either dimension, the PSD estimate must suffer spectral leakage (Harris, 1978), because Earth surfaces return power at a continuum of ranges and along-track positions, and not only those corresponding to the discrete frequencies that can be correctly sampled by the DFT. This paper is about designing a spectral window sequence to mitigate that spectral leakage, in either or both dimensions.

The function characterizing the resolution of the PSD estimate is known in the radar literature as the point target response (PTR). A spectral window shapes the PTR. Theoretical expressions for the statistical expectation of an altimeter's range PSD over a rough surface assume a Gaussian PTR (Berger, 1972; Brown, 1977; Wingham et al. 2004; Ray et al., 2015). To approximate a Gaussian shape, the ERS-1, ERS-2, and EnviSat altimeters employed a Hamming window in range (Francis, 1986; Quartly et al.,

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2001; Roca et al., 2009). Poseidon-heritage altimeters (Poseidon, Jason-1, Jason-2, Jason-3, CryoSat-2, Sentinel-3) use a rectangular window in range (Armand et al., 1998; Amarouche et al. 2004), with a Hamming window used in the along-track dimension for the D/D-SAR mode of CryoSat-2 (European Space Agency, 2012).

During meetings of the CryoSat-2 Quality Working Group (QWG) it became clear that there are a number of confusing issues in the correct specification of spectral windows for satellite altimeters. Surface roughness estimation requires correct characterization of the PTR width, but the altimeter literature has conflicting values (see discussion and references cited in the paragraph following Eq. 3 in Amarouche et al., 2004). There are a variety of specifications of the Hamming window in various radar textbooks (demonstrated below in Section 4) and these are sufficiently diverse to have caused QWG members to obtain conflicting results when analyzing the effect of the window.

This paper will (1) clarify the correct sampling, symmetry and zero-padding of windows needed in altimetry; (2) propose that PTR characterization should use measures defined by integrals of the PTR, since the PTR influences the waveform through a convolution; (3) suggest an integral measure characterizing how closely the PTR behaves like a Gaussian under convolution; (4) show that integral measures naturally lead to optimal windows expressed as eigenvectors; and (5) suggest that windows other than the Hamming can achieve adequate leakage suppression with narrower main-lobe resolution and more nearly Gaussian performance under convolution. Theory is presented generically. Applications are illustrated with CryoSat-2 data.

2. Fast, slow, and non-dimensional time

Altimeters employ "full deramp" of a linear frequency-modulated (FM) chirp to synthesize pulse compression (MacArthur, 1976), by means of which the frequency in each processed pulse echo is proportional to range (Chelton et al., 1989). The frequency band of interest, approximately 1 percent of the chirp bandwidth, or a few MHz, passes through an analog anti-aliasing filter, after which the ADC samples each echo at a rate sufficient to prevent aliasing (Nyquist-Shannon sampling theorem; Nyquist, 1928; Shannon, 1948). The ADC in the "SIRAL" altimeter on board CryoSat-2 produces $N_1 = 128$ complex (in-phase, I, and quadrature, Q) samples from each echo at a sampling interval $\Delta t_1 = 350$ ns. The PSD of this sequence reveals backscattered power as a function of range. When PSD estimation is along this dimension only, the CryoSat literature calls this the low-resolution mode (LRM) or pulse-limited waveform (Wingham et al., 2006).

Motion of the spacecraft during pulse transmission and echo reception introduces Doppler frequencies of a few kHz that depend on the along-track position of scatterers (Raney, 1998). D/D-SAR processing applies a DFT across a sequence of consecutive echoes called a "burst" to create a fan of Doppler-sharpened beams. The PSD of this sequence maps backscattered power to along-track position (Raney, 1998; Wingham et al., 2006). The (within-burst) pulse repetition interval, Δt_2, and the number of pulses in a burst, N_2, provide the sampling of this dimension of time. CryoSat-2's "SAR" mode has $N_2 = 64$ and $\Delta t_2 = 55\ \mu\text{s}$.

Here, t_1 indicates time within an echo, and t_2 indicates time from echo to echo. The frequencies of interest in the two dimensions differ by about two orders of magnitude, so they may be called "fast time" and "slow time" (see p. 143 in Cumming and Wong, 2004). Fast time reveals chirp frequency and range; slow time reveals Doppler frequency and along-track position. In order to apply to either dimension or both, equations in this paper will use a non-dimensional time, t, and frequency, f, normalized so that $\Delta t = 1$; the number of samples in a time sequence will be N, or M if the sequence is zero-padded (see Section 5). A time sequence will be indicated $\{x\}$, with x_k indicating an individual sample. Sample indices will run from 0 to $N-1$, or $M-1$ in zero-padded sequences.

3. Windows and spectral leakage

High-fidelity analog-to-digital conversion is a subject in its own right (see Chen et al. (2016) and references therein) but for purposes of this paper suppose that a length-N data sequence $\{x\}$ samples a (complex, in general) continuous time process $x(t)$ so that $x_k = x(k)$ for k from 0 through $N-1$. A windowed data sequence $\{y\}$ has elements $y_k = x_k w_k$, where $\{w\}$ is a real-valued spectral window sequence; if no window is used, $w_k = 1$ for all k, called here a "URN" for unit rectangle of length N. The Fourier transform of $x(t)$ that would be obtained by an integral over all time would be $X(f)$, but operations on $\{y\}$ can only estimate the function $Y(f) = X(f) * W(f)$, where the asterisk indicates convolution over the entire frequency axis, and

$$W(f) = \sum_{k=0}^{N-1} w_k \exp(-2\pi f k \sqrt{-1}) \tag{1}$$

is the discrete time Fourier transform of $\{w\}$. $W(f)$ is a continuous function of frequency defined for all f and periodic with period 1, so it suffices to consider the Nyquist interval $|f| \leqslant 1/2$; if $x(t)$ has been properly band-limited before it is sampled, then the convolution is effectively a circular convolution over the Nyquist interval. Spectral leakage is the phenomenon that $Y(f) \neq X(f)$ when N is finite. Window "carpentry" (the term coined by Tukey (1961) at p. 214 for the shaping of windows to achieve desired ends) must balance conflicting objectives for the fidelity of $Y(f)$ to $X(f)$.

4. An example illustrating the confusion around sampling, parity and symmetry

Many windows in the literature are defined by a continuous function $w(t)$ defined on a time interval of length T

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