ARTICLE IN PRESS

Available online at www.sciencedirect.com

ScienceDirect

Advances in Space Research xxx (2018) xxx–xxx

ELSEVIER

Advances in Space Research (a COSPAR publication)

www.elsevier.com/locate/asr

An optimization approach for observation association with systemic uncertainty applied to electro-optical systems

Johnny L. Worthy III [a,*,1], Marcus J. Holzinger [a,2], Daniel J. Scheeres [b,3]

[a] Georgia Institute of Technology, Atlanta, GA 30332, United States

[b] The University of Colorado Boulder, Boulder, CO 80305, United States

Received 20 December 2016; received in revised form 17 February 2018; accepted 28 February 2018

Abstract

The observation to observation measurement association problem for dynamical systems can be addressed by determining if the uncertain admissible regions produced from each observation have one or more points of intersection in state space. An observation association method is developed which uses an optimization based approach to identify local Mahalanobis distance minima in state space between two uncertain admissible regions. A binary hypothesis test with a selected false alarm rate is used to assess the probability that an intersection exists at the point(s) of minimum distance. The systemic uncertainties, such as measurement uncertainties, timing errors, and other parameter errors, define a distribution about a state estimate located at the local Mahalanobis distance minima. If local minima do not exist, then the observations are not associated. The proposed method utilizes an optimization approach defined on a reduced dimension state space to reduce the computational load of the algorithm. The efficacy and efficiency of the proposed method is demonstrated on observation data collected from the Georgia Tech Space Object Research Telescope.

Keywords: Space situational awareness; Observation association; Admissible regions

1. Introduction

The association of uncorrelated observations is an open area of research in space situational awareness (SSA). The purpose of SSA is to catalog as fully as possible the space environment, which falls under the more broad space domain awareness (SDA) which aims to characterize and surveil the space environment (JP3, 2013; Blake et al., 2012). In particular, this entails using observations from measurement systems to estimate the state of an object. The difficulty in this task is that, given a single sensor, two or more observations are required to generate candidate state solutions consistent with the dynamics of the system. Performing observation to observation association is necessary to identify which observations may be used together to produce or refine a state estimate. This paper develops a methodology by which to probabilistically associate observations and identify potential state solutions from correlated observations.

The observation of underdetermined dynamical systems often results in certain states that cannot be determined from a single observation. SSA and SDA are directly impacted by this problem due to types of sensors used to detect space objects. Optical sensors measure the bearing to an object, and over time the bearing rates can be derived, however, range and range-rate are unmeasured. Radar sensors measure the range, range-rate, and bearing to an

* Corresponding author.

E-mail addresses: johnny.worthy@ll.mit.edu (J.L. Worthy III), holzinger@gatech.edu (M.J. Holzinger), scheeres@colorado.edu (D.J. Scheeres).

[1] Graduate Researcher, School of Aerospace Engineering.

[2] Assistant Professor, School of Aerospace Engineering.

[3] A. Richard Seebass Chair Professor, Department of Aerospace Engineering Sciences.

https://doi.org/10.1016/j.asr.2018.02.041

Please cite this article in press as: Worthy, J.L., et al. An optimization approach for observation association with systemic uncertainty applied to electro-optical systems. Adv. Space Res. (2018), https://doi.org/10.1016/j.asr.2018.02.041

object, but the bearing rates are unmeasured. Both radar and optical sensors are unable to uniquely identify all the states of a space object given a single measurement over a short period of time relative to the orbit of the object. Over long observation periods with multiple independent measurements, a full state estimate is traditionally produced from these sensors via approaches such as Gauss' method or Lambert's method (Taff, 1984). It can be shown that over sufficiently short time periods, the dynamics of this observation problem are essentially unobservable (Worthy and Holzinger, 2015). This unobservability leads to approaches such as Gauss' or Lambert's methods suffering from numerical difficulties, as there exists a continuum of possible state solutions, and an alternative solution approach must be utilized.

The admissible region utilizes hypothesized constraints to bound the continuum of consistent state solutions in the unobservable space of a sequence of measurements (Milani et al., 2004). This approach, proposed by Milani et al., hypothesizes constraints which bound a set of feasible solutions in the unobservable, or undetermined, state space (Milani et al., 2004, 2005). Since the introduction of the admissible region, much work has gone into improving the admissible region approach by representing the admissible region probabilistically. Fujimoto and Scheeres (2012) show that points contained within the admissible region necessarily have a uniform distribution since no state is more likely than another to be the true state. This results in a probability density discontinuity at the boundary of the admissible region since the constraint is binary. However, if uncertainty in the measurement and observer parameters are included then this probability discontinuity is eliminated and instead a continuous probability of set membership describes the probability distribution over the admissible region. This is demonstrated in an approximate analytical approach by Worthy and Holzinger (2015) which directly maps measurement error and observer parameter uncertainty into the definition of the admissible region to define a continuous probability of set membership. This approximate analytic approach is shown to be well in agreement with a full nonlinear Monte-Carlo simulation. DeMars and Jah (2013) use a Gaussian mixture model to probabilistically represent the admissible region in a computationally efficient way. Hussein et al. (2014a) addresses uncertainty in the observation by obtaining the statistics of the determined states from the observation using a least squares approach and uniformly samples a subset of orbital elements representing the undetermined states to define a full state. Then each full state is mapped into the undetermined state space to produce a probabilistic admissible region probability distribution by fitting a Gaussian mixture to the resulting points.

In general, while the admissible region approach generates a probability density function (PDF) over candidate solutions, it alone does not directly enable an association of two disparate observations of an object. The association of the observations of underdetermined systems is another open area of research. Much work has been done to extend the admissible region approach to allow for association of observation of space objects. Milani et al. (2011) gives an overview of two object association methods. The first method uses an attribution penalty defined over the observable, or determined, states that is computed for each sampled point in an admissible region to identify potentially correlated objects (Milani and Gronchi, 2010). If a given point has an attribution penalty less than a maximum allowed penalty then that point is a candidate for association. The second method leverages integrals of motion by requiring that the energy and angular momentum for each disparate observation are equal. Maruskin et al. (2009) shows that a set of points from two admissible regions at a common epoch mapped to 6-D Delaunay element space must necessarily intersect for the observations to be correlated. Fujimoto and Scheeres (2012) show that an admissible region mapped to 6-D Poincaré element space can be assigned a probability distribution and combined with other observations by using Bayes' theorem. It is shown that if an intersection exists in Poincare elements, then not only are the observations correlated but also the intersection is the solution. Furthermore, it is mentioned that the limiting case of this approach is when the time between observations becomes exceedingly large relative to the orbit period as multiple intersection solutions are possible. Fujimoto et al. (2014) extend this method with a hybrid direct Bayesian and least squares approach to demonstrate object association. Hussein et al. (2014b) utilize mutual information and information divergence, both information theoretic methods, to compare the distributions of a set of measurements to determine if the measurements are associated.

The association approach proposed in this paper will expand on the admissible region intersection based methods introduced above to demonstrate the correlation of disparate observations utilizing an optimization based approach. This optimization based approach is similar to the optimization approach proposed by Siminksi et al. where a loss function is optimized to identify potential orbit state solutions, which is equivalent to finding the intersection of the admissible regions (Siminski et al., 2014). However, in the presence of uncertainty the location of the intersection may not occur at a single point, but over a set of possible intersection points, generating local minima rather than a well defined point of intersection. As such, there should be a probability of association metric that determines which local minimum is most likely the true state. The work shown in this paper develops a methodology to properly estimate the PDF about these local minima from which the PDF of the initial state estimates for the observation can be determined.

The contributions of the paper are outlined as follows: (1) an optimization framework to identify the local Mahalanobis distance minima of n dimensional spaces using multiple d dimensional subspaces applied to optical observations using a general distance metric, (2) a binary

Please cite this article in press as: Worthy, J.L., et al. An optimization approach for observation association with systemic uncertainty applied to electro-optical systems. Adv. Space Res. (2018), https://doi.org/10.1016/j.asr.2018.02.041