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Convex optimisation approach to constrained fuel optimal control of spacecraft in close relative motion

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Abstract

This paper describes an interesting and powerful approach to the constrained fuel-optimal control of spacecraft in close relative motion. The proposed approach is well suited for problems under linear dynamic equations, therefore perfectly fitting to the case of spacecraft flying in close relative motion. If the solution of the optimisation is approximated as a polynomial with respect to the time variable, then the problem can be approached with a technique developed in the control engineering community, known as "Sum Of Squares" (SOS), and the constraints can be reduced to bounds on the polynomials. Such a technique allows rewriting polynomial bounding problems in the form of convex optimisation problems, at the cost of a certain amount of conservatism. The principles of the techniques are explained and some application related to spacecraft flying in close relative motion are shown.

Keywords: Fuel optimal control; Constrained optimal control; Close relative motion; Convex optimisation; Sum of squares

1. Introduction

The search for fuel-optimal manoeuvres is a classical problem in space engineering (Scharf et al., 2003), which is still thoroughly investigated by the aerospace community in search of more efficient and reliable methods, for different mission profiles (Li, 2016; Bolle and Circi, 2012; Qi and Jia, 2012). The problem is of critical interest due to the hard constraints on the quantity of fuel (and consequently, of delta-v) that a spacecraft can carry at launch. The classical analytical approach is based on Pontryagin's principle, which yields the classical bang-off-bang solutions (Kirk, 2012). Nevertheless, closed form solutions of fuel-optimal problems are often impossible to find, which makes it necessary the use of numerical optimisation methods.

The numerical solution of the optimal control problem, which is central to the fuel-optimal problem, can be found in two different ways, using indirect methods or direct methods. Indirect methods are based on the writing of the Hamiltonian function and on the solution of the Euler-Lagrange differential equation. In general they lead to very accurate results with the use of few variables. On the contrary, direct methods are based on the transcription of the differential problem into a pure parametric problem which can be solved using direct optimization methods. This kind of methods can lead to solutions as accurate as indirect methods but requires the use of many more variables. In both cases, the discrete problem can be faced with the algorithms developed for parameter optimization which are typically based on the Newton method (Betts, 1998). Example of indirect methods can be seen in Casalino et al. (1999) and Zhang et al. (2015), while

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example of direct methods can be seen in Massari and Bernelli-Zazzera (2009) and Massari et al. (2003).

In general, both indirect and direct methods are very powerful, but being based on the Newton method, they require an initial solution guess to start the iterations. Moreover, this solution should be near enough to a local minimum to guarantee the convergence of the method to a solution. This shows also a second drawback of those methods, only local minima can be reached, no information on the globality of the optimum can be achieved.

The method presented in this paper belongs to the class of convex optimisation based methods, as do those based on Linear Programming (LP) (Magnani and Boyd, 2009) and moment measures (Claeys et al., 2014), which have also been applied to the problems described above. In this article, we explore an approach based on a technique known as Sum Of Squares (SOS) (Parrilo, 2003), which lets one formulate polynomial optimisation problems in the form of a convex optimisation without any need of discretising the dynamical equations. With this technique, assuming that the solution has a polynomial expression, the problem can be cast into the form of an optimisation under Linear Matrix Inequality (LMI) constraints or Semi-Definite Programming (SDP), a form of convex optimisation that has been developed in the last decades in the context of automatic control (Boyd et al., 1994). The interest of this method is that it turns the problem into a convex one, in a very direct and simple way which is easily understandable even for the non-experts of the specific optimisation techniques involved. For this reason, this paper has also an introductory or tutorial part which allows a better understanding of the fundamentals.

As it will be explained later on, the reformulation of the problem required by the technique is done at the cost of a loss of precision, but on the other hand, the convex formulation does not require any initial guess, and it does not feature the risk of yielding local optima. The proposed technique clearly brings advantages with respect to classical indirect or direct approach to the solution of optimal control problems.

The paper is organised as follows. Section 2 introduces and formulates the problem. Sections 3 and 4 contain a short tutorial for explaining the ideas behind Sum Of Squares (SOS) and Linear Matrix Inequalities (LMIs) techniques, which we think improve the readability of this paper, but they can be skipped by those who are already familiar. Section 5 contains the baseline algorithm that is the main result of this article, whereas Section 6 introduces a few variants on it. Section 7 shows a set of application to spacecraft in close relative motion and finally Section 8 draws the conclusions.

1.1. Notation

We denote by $\mathbb{N}$ the set of non-negative integers, by $\mathbb{R}$ the set of real numbers and by $\mathbb{R}^{n\times m}$ the set of real $n \times m$ matrices. $\mathbf{R}_m[x]$ is the set of real-valued polynomials of degree m in the entries of x, $A^\top$ indicates the transpose of a matrix A; the notation $A \succeq 0$ (resp. $A \preceq 0$) indicates that all the eigenvalues of the symmetric matrix A are positive (resp. negative) or equal to zero. The symbol $\binom{n}{k}$ indicates the binomial coefficient, for which we have

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}.$$

For the reader's convenience, all the symbols of this paper with exception of those used in the examples are listed at the end in the Appendix.

2. Problem formulation

We consider linear dynamic equations describing the motion of one or more point masses, of the kind

$$\ddot{x}(t) = f(x(t)) + u(t) \tag{1}$$

where t is the time variable, $x(t) = [x_1(t), \ldots, x_n(t)]^\top \in \mathbb{R}^n$ the position vector (with $n \in \mathbb{N}$), $u(t) = [u_1(t), \ldots, u_n(t)]^\top \in \mathbb{R}^n$ a vector of control actions and $f(x(t)) = [f_1(x(t)), \ldots, f_n(x(t))]^\top$ a vector-valued linear function coming from the physics of the problem. The typical fuel-optimal problem consists in finding a trajectory $x^*(t)$ which brings the state from an initial position x_0 and velocity v_0 at time $t = 0$, to a final position x_f and velocity v_f at a fixed time t_f, minimising the time integral of a one-norm of $u^*(t) = \ddot{x}^*(t) - f(x^*(t))$. This can be formulated formally as follows.

Problem 1 (*Fuel-optimal control*). Given (1), $t_f > 0$, $u_{max,i} > 0, x_0, v_0, x_f, v_f$, find a continuous and derivable function $x^*(t) : [0, t_f] \mapsto \mathbb{R}^n$ such that

$$\int_0^{t_f} \sum_i |u_i^*(t)|\, dt \text{ is minimised} \tag{2}$$

under $x^*(0) = x_0, \dot{x}^*(0) = v_0,\ x^*(t_f) = x_f, \dot{x}^*(t_f) = v_f, |u_i^*(t)| \leqslant u_{max,i}$, with $u_i^*(t) = \ddot{x}_i^*(t) - f_i(x^*(t))$.

Notice that by setting one of the $u_{max,i}$ as very small or close to zero, one can take into account situations where not all the directions of the space are directly actuated, i.e. the cases in which $u_i(t) = 0$ for a few (not all) values of i.

The methods discussed in this paper cannot deal directly with Problem 1, but rather with a relaxation of it. By "relaxing a problem", we mean replacing the original problem with a second one that converges to the first under certain hypotheses. The advantage of doing so is that the second problem is amenable to a new approach, and it is formulated as follows.

Problem 2 (*Relaxed fuel-optimal control*). Given (1), $t_f > 0, u_{max,i} > 0, x_0, v_0, x_f, v_f, N \in \mathbb{N}, d \in \mathbb{N}$, find a piecewise-polynomial vector-valued function $x^*(t) : [0, t_f] \mapsto \mathbb{R}^n$ defined as