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Model of a fluxtube with a twisted magnetic field in the stratified solar atmosphere

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Abstract

We build a single vertical straight magnetic fluxtube spanning the solar photosphere and the transition region which does not expand with height. We assume that the fluxtube containing twisted magnetic fields is in magnetohydrostatic equilibrium within a realistic stratified atmosphere subject to solar gravity. Incorporating specific forms of current density and gas pressure in the Grad–Shafranov equation, we solve the magnetic flux function, and find it to be separable with a Coulomb wave function in radial direction while the vertical part of the solution decreases exponentially. We employ improved fluxtube boundary conditions and take a realistic ambient external pressure for the photosphere to transition region, to derive a family of solutions for reasonable values of the fluxtube radius and magnetic field strength at the base of the axis that are the free parameters in our model. We find that our model estimates are consistent with the magnetic field strength and the radii of Magnetic bright points (MBPs) as estimated from observations. We also derive thermodynamic quantities inside the fluxtube.

Keywords: Magnetohydrostatics; Sun: photosphere; Sun: transition region; Sun: magnetic fields; Magnetohydrodynamics (MHD)

1. Introduction

The study of small scale magnetic structures in the solar photosphere is important because they play a crucial role in the evolution of active regions and sunspots (Muller and Mena, 1987; Centeno et al., 2007). Magnetic bright points (MBPs) are likely to be the fluxtubes observed in the photosphere (Berger et al., 1995; Centeno et al., 2007; Lagg et al., 2010). The topological rearrangement of these magnetic fluxtubes due to the motion of the photospheric foot points or magnetic reconnections, contribute to the coronal heating (Muller et al., 1994; van Ballegooijen, 1986). A three dimensional (3D) single fluxtube model with untwisted magnetic field has been studied by solving linear elliptic partial differential equation by numerical iterative process (Steiner et al., 1986). Schlüter and Temesváry (1958) and Osherovich (1984) studied a 3D fluxtube for sunspots using a self–similar model. The magnetic and thermodynamic structure inside fluxtube with untwisted magnetic field which spans from photosphere to the lower part of the solar corona is studied by Gent et al. (2013). Both 2D and 3D numerical models of fluxtubes with the energy propagation through the torsional Alfven waves have been studied by Murawski et al. (2015a,b), where an empirical form of magnetic flux function was assumed. Vigeesh et al. (2009) assumed an empirical form of gas pressure to investigate the wave propagation and energy transport through a fluxtube. Several interesting results of wave behavior in the solar photosphere and chromosphere have been presented by several authors (Bogdan et al., 2003; Fedun et al., 2009; Shelyag et al., 2010).

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In this work, we construct a 3D single cylindrical vertical straight magnetic fluxtube semi-analytically with a twisted magnetic field by obtaining a new solution of poloidal flux function by solving Grad–Shafranov equation (GSE; Grad and Rubin, 1958; Shafranov, 1958). We assume a specific form of gas pressure and poloidal current, which has been used to study the equilibrium solution of terrestrial plasma (Atanasiu et al., 2004). An equilibrium solution near the magnetic axis of the plasma torus has been reported previously, using a plasma pressure and poloidal current profile that varies linearly with the poloidal flux function (Solov'ev, 1968). We obtain an analytic solution by assuming a form that is quadratic in the poloidal flux function, and derive the magnetic field structure and thermodynamic quantities inside the fluxtube using the solution that represents an ideal MHS equilibrium. In the future, we will look to explore fully the profile functions that will improve the solution set.

The overview of the paper is as follows. In Section 2, the GSE has been derived assuming a specific form of the profile function of gas pressure and poloidal current and the solution of the equation is presented. In Section 3, we discuss the boundary condition that is physically acceptable, and can be used for realistic modelling of a fluxtube. In Section 4, the mode wise variation of the profile functions are presented and in Section 5, we compare the model with the observations. Finally, we conclude with a comparison with other existing models.

2. Solution of Grad-Shafranov equation

We assume an axisymmetric cylindrical geometry, with gas pressure p and take the poloidal current I_p constant along a magnetic field line. We express $p(\Psi, z)$ and $I_p(\Psi)$ in terms of the poloidal flux function $\Psi(r,z)$ and z and consider a straight vertical axisymmetric fluxtube that spans the altitude from photosphere ($z=0$) to the transition region ($z = 2.15$ Mm) that is in equilibrium with the atmosphere outside with the uniform gravity $\mathbf{g}(=-g\hat{z})$ acting vertically downward. The force balance equation in MHS equilibrium takes the form

$$-\nabla p + \frac{1}{4\pi}(\nabla\times\mathbf{B})\times\mathbf{B} + \rho\mathbf{g} = 0, \tag{1}$$

where ρ denotes the mass density and $\mathbf{B}$ is the magnetic field associated with the poloidal flux function $\Psi(r,z) = \int_0^r B_z(r',z) r' \mathrm{d}r'$ (scaled by the factor $\frac{1}{2\pi}$) in the following form

$$B_r = -\frac{1}{r}\frac{\partial\Psi}{\partial z}; \quad B_z = \frac{1}{r}\frac{\partial\Psi}{\partial r}; \quad B_\phi = \frac{I_p}{r}. \tag{2}$$

This form of B_r, B_ϕ and B_z ensures the solenoidal condition of magnetic field. Now splitting the MHS force balance Eq. (1) into r and z directions, we find two different scalar partial differential equations

$$-\frac{\partial p}{\partial r} + \frac{1}{4\pi}\left(B_r\frac{\partial B_r}{\partial r} + B_z\frac{\partial B_r}{\partial z} - \frac{1}{2}\frac{\partial B^2}{\partial r}\right) = 0 \tag{3a}$$

$$-\frac{\partial p}{\partial z} + \frac{1}{4\pi}\left[\frac{1}{r}\frac{\partial\Psi}{\partial z}\left(\frac{1}{r^2}\frac{\partial\Psi}{\partial r} - \frac{1}{r}\frac{\partial^2\Psi}{\partial r^2}\right) - \frac{1}{r^2}\frac{\partial\Psi}{\partial z}\frac{\partial^2\Psi}{\partial z^2} - \frac{1}{2r^2}\frac{\partial}{\partial z}(I_p^2)\right] - \rho g = 0. \tag{3b}$$

If the gas pressure and poloidal current are functions of Ψ alone i.e., $p_1(\Psi)$ and $I_p(\Psi)$ respectively, then from the (3a, 2) it follows that

$$\frac{\partial^2\Psi}{\partial r^2} - \frac{1}{r}\frac{\partial\Psi}{\partial r} + \frac{\partial^2\Psi}{\partial z^2} = -\frac{1}{2}\frac{\partial I_p^2(\Psi)}{\partial\Psi} - 4\pi r^2\frac{\partial p_1(\Psi)}{\partial\Psi}. \tag{4}$$

Plugging in p_1 and I_p^2 in (3b), we find

$$-\frac{\partial p_1(\Psi)}{\partial z} + \frac{1}{4\pi}\left[\frac{1}{r^2}\frac{\partial\Psi}{\partial z}\left(\frac{1}{r}\frac{\partial\Psi}{\partial r} - \frac{\partial^2\Psi}{\partial r^2}\right) - \frac{1}{r^2}\frac{\partial\Psi}{\partial z}\frac{\partial^2\Psi}{\partial z^2} - \frac{1}{2r^2}\frac{\partial I_p^2(\Psi)}{\partial z}\right] - \rho g = 0. \tag{5}$$

By multiplying both sides of (5) by $4\pi r^2\frac{\partial z}{\partial\Psi}$ and using (4), we obtain $g\rho\frac{\partial z}{\partial\Psi} = 0$ which implies that ρ is zero, which means that the vertical hydrostatic pressure balance will not be maintained. Therefore, to balance the vertical hydrostatic pressure inside the fluxtube, we introduce a new function, $p_2(z)$ such that,

$$p(r,z) = p_1(\Psi) + p_2(z).$$

We assume $p_1(\Psi)$ and $I_p^2(\Psi)$ to be second order polynomials of Ψ

$$p(r,z) = p_1(\Psi) + p_2(z) \tag{6a}$$

$$I_p^2(r,z) = \tilde{\alpha}\Psi^2 + \tilde{\beta}\Psi + I_0^2 \tag{6b}$$

where

$$p_1(\Psi) = \tilde{a}\Psi^2 + \tilde{b}\Psi,$$

and the parameters $\tilde{a}, \tilde{b}, \tilde{\alpha}, \tilde{\beta}$ and I_0^2 are to be determined by appropriate boundary conditions. The function $p_2(z)$ is to be evaluated later. The substitution of p given by (6a) in (3a) gives (4) and we obtain the following second order scalar partial linear inhomogeneous differential equation

$$\frac{\partial^2\Psi}{\partial r^2} - \frac{1}{r}\frac{\partial\Psi}{\partial r} + \frac{\partial^2\Psi}{\partial z^2} = -(ar^2+\alpha)\Psi - (br^2+\beta), \tag{7}$$

with the rescaled parameters, $a = 8\pi\tilde{a}; \alpha = \tilde{\alpha}; b = 4\pi\tilde{b}; \beta = \tilde{\beta}/2$. To solve (7), we split Ψ in two parts: a homogeneous part, $\Psi_h(r,z)$ and an inhomogeneous part $\Psi_p(r)$, i.e. $\Psi(r,z) = \Psi_h(r,z) + \Psi_p(r)$. Using this form in (7), we separate the homogeneous and the inhomogeneous parts to obtain the following expressions:

$$\frac{\partial^2\Psi_h}{\partial r^2} - \frac{1}{r}\frac{\partial\Psi_h}{\partial r} + \frac{\partial^2\Psi_h}{\partial z^2} = -(ar^2+\alpha)\Psi_h \tag{8a}$$

$$\frac{\partial^2\Psi_p}{\partial r^2} - \frac{1}{r}\frac{\partial\Psi_p}{\partial r} = -(ar^2+\alpha)\Psi_p - (br^2+\beta). \tag{8b}$$

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