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# Fundamental (*f*) oscillations in a magnetically coupled solar interior-atmosphere system – An analytical approach

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## Abstract

Solar fundamental (*f*) acoustic mode oscillations are investigated analytically in a magnetohydrodynamic (MHD) model. The model consists of three layers in planar geometry, representing the solar interior, the magnetic atmosphere, and a transitional layer sandwiched between them. Since we focus on the fundamental mode here, we assume the plasma is incompressible. A horizontal, canopy-like, magnetic field is introduced to the atmosphere, in which degenerated slow MHD waves can exist. The global (*f*-mode) oscillations can couple to local atmospheric Alfvén waves, resulting, e.g., in a frequency shift of the oscillations. The dispersion relation of the global oscillation mode is derived, and is solved analytically for the thin-transitional layer approximation and for the weak-field approximation. Analytical formulae are also provided for the frequency shifts due to the presence of a thin transitional layer and a weak atmospheric magnetic field. The analytical results generally indicate that, compared to the fundamental value ($\omega = \sqrt{gk}$), the mode frequency is reduced by the presence of an atmosphere by a few per cent. A thin transitional layer reduces the eigen-frequencies further by about an additional hundred microhertz. Finally, a weak atmospheric magnetic field can slightly, by a few percent, increase the frequency of the eigen-mode. Stronger magnetic fields, however, can increase the *f*-mode frequency by even up to ten per cent, which cannot be seen in observed data. The presence of a magnetic atmosphere in the three-layer model also introduces non-permitted propagation windows in the frequency spectrum; here, *f*-mode oscillations cannot exist with certain values of the harmonic degree. The eigen-frequencies can be sensitive to the background physical parameters, such as an atmospheric density scale-height or the rate of the plasma density drop at the photosphere. Such information, if ever observed with high-resolution instrumentation and inverted, could help to gain further insight into solar magnetic structures by means of solar magneto-seismology, and could provide further insight into the role of magnetism in solar oscillations.

*Keywords:* Helioseismology; F mode; Solar interior; Solar atmosphere; Magnetic field; MHD

## 1. Introduction

The fundamental mode or *f* mode is a surface oscillation radially localised right beneath the solar photosphere in solar interior models (Christensen-Dalsgaard, 2002; Christensen-Dalsgaard et al., 2005). In plane-wave approximation with a free photosphere with no atmosphere above it in a standard solar model, the cyclic frequency of the *f* mode is $\nu = (2\pi)^{-1}\sqrt{gk}$. Here, $g$ is the gravitational acceleration and $k$ is the wavenumber of the oscillation mode, both taken at the photosphere. Measuring the *f* mode frequency is an effective diagnostic method for determining the near-photospheric physical features, like density or plasma flows. Among others, it also provides an accurate measure of the solar radius (Antia et al., 2000).

Observations show that helioseismic *f* and *p* (pressure) modes are effected by the presence of magnetic structures

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in the solar environment. By measuring the variations in properties of wave modes, we can probe the environment of the waves. This can be subsurface structures of magnetic active regions (Liang et al., 2013; Zhao and Chou, 2016) or the lower atmospheric layers above the photosphere. Both inhomogeneities (e.g., density, temperature or magnetic) and dynamic changes (e.g., random or coherent flows) in the atmosphere can modify *f* and *p* mode properties (see e.g. Erdélyi et al., 2005; Mole et al., 2008; Kerekes et al., 2008a,b). Frequencies (Libbrecht and Woodard, 1990; Chaplin et al., 2004; Dziembowski and Goode, 2005; Tripathy et al., 2006; Chaplin et al., 2007) and line widths (Tripathy et al., 2006; Lamb, 1932; Tripathy et al., 2006) of *f* and *p* oscillation modes vary in time and they correlate with the solar activity cycle (see also the review papers by Erdélyi (2006a,b) Thompson, 2006). The observed magnetic effects on the helioseismic oscillations indicate that a possible connection between global *f*- and *p*-mode oscillations and the magnetic solar atmosphere. Solar atmospheric properties influence the helioseismic oscillations and as such the oscillation characteristics vary together with the variations of the atmospheric condition. Our goal is to pursue this line of investigations by considering mathematically a coupled solar interior-atmosphere system as one single physical entity.

Considerable efforts have already been made to interpret the observed variations of the properties of global helioseismic modes. Possible effects of an atmospheric magnetic field on global solar oscillations were investigated first by Campbell and Roberts (1989) in a magnetohydrodynamic (MHD) model. In Cartesian geometry, they considered a two-layered model with a polytropic interior and an overlying atmosphere with constant temperature embedded in a horizontal magnetic field with constant Alfvén speed. They found that the magnetic field increases the frequency of the *f* mode. A series of other models followed that of Campbell and Roberts (1989). Various aspects of the global oscillations of the coupled solar interior and atmosphere were studied. Global waves propagating obliquely to the horizontal magnetic field have been investigated by Jefferies et al. (1990) and Pintér (2008). The effects of atmospheric temperature profiles have been studied by Vanlommel and Čadež (1998). Taroyan (2004) and Taroyan et al. (2004) have investigated effects of plasma flows on MHD waves. Random horizontal and vertical plasma flows both influence the *f*-mode properties (Mole et al., 2008; Kerekes et al., 2008a,b). Stochastic atmospheric magnetic fields are modelled by Erdélyi et al. (2005). In order to model the damping of global oscillations, the resonant coupling of these oscillations to atmospheric local slow and Alfvén waves was proposed and investigated in the context to contribute to the heating of the solar corona. This phenomenon is introduced in Sakurai et al. (1991), Zhukov (1997) and Tirry et al. (1998), and has been further explored by Pintér (1999), Pintér et al. (2007) and Taroyan and Erdélyi (2008a,b). Pintér and Erdélyi (2011) is an overview of modelling magnetic atmospheric effects on helioseismic oscillations.

In the present paper, we advance a coupled solar interior-atmosphere model in which magnetic atmospheric effects on *f*-mode frequencies due to the coupling of the interior to the atmosphere are studied in a three-layered model accounting for the interior, magnetic atmosphere and a transitional layer sandwiched in-between. An analytical form of the dispersion relation, $f(\omega, k) = 0$, is provided. The *f*-mode frequency is determined from the dispersion equation for two distinct cases. One considers an approximation with a solar interior and a single-layer magnetic atmosphere. In the other model, the magnetic field is turned off and a transitional layer is added to the lower part of the solar atmosphere representing e.g. the chromospheric canopy, in which the equilibrium parameters of the plasma vary continuously. Analytical formulae are derived for the frequency shift due to the presence of a thin transitional layer at the bottom of a magnetic atmosphere and, separately, due to the presence of a weak magnetic field in an atmosphere itself with a transitional layer.

All the earlier studies, e.g. those cited above, have used numerical methods to obtain a frequency versus wavenumber relationship, and their frequency spectra can only be presented graphically. The analytical approach in this study offers a clear complementary view of the influence of various plasma parameters, such as an atmospheric density scale-height, on the *f*-mode frequency. This may increase the diagnostic value of helioseismic observations as frequency shifts can be explicitly scaled with e.g. magnetisms or parameters determining stratification.

Our ultimate aim is to find, analytically, the consequences of resonant coupling between the solar interior and the inhomogeneous magnetised solar atmosphere. The study here is only the first step as we only focus on the incompressible mode of the interior-atmosphere system. We wish to shed light on the underlying physics before the complicated phenomenon of resonance is introduced in a follow-up study.

The simplified MHD model of the Sun is described in Section 2. A dispersion relation is derived analytically from the governing equations in Section 3. The dispersion relation is solved analytically for representative cases. The basic working model, described in Section 4, is a two-layer model with no magnetic field in the atmospheric layer. Next, a weak-field approximation is discussed in Section 5, where magnetism is introduced in the solar atmosphere, while Section 6 presents results of studies to obtain the frequencies of *f*-mode oscillations in a model with a thin transitional layer in the lower atmosphere, where the magnetic field strength increases continuously from zero with increasing height. We conclude in Section 7.

## 2. The equilibrium

The Sun is modelled by a plane-parallel, three-layer plasma, using Cartesian geometry. One layer is the non-