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Uncertainty propagation for statistical impact prediction of space debris

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Abstract

Predictions of the impact time and location of space debris in a decaying trajectory are highly influenced by uncertainties. The traditional Monte Carlo (MC) method can be used to perform accurate statistical impact predictions, but requires a large computational effort. A method is investigated that directly propagates a Probability Density Function (PDF) in time, which has the potential to obtain more accurate results with less computational effort. The decaying trajectory of Delta-K rocket stages was used to test the methods using a six degrees-of-freedom state model. The PDF of the state of the body was propagated in time to obtain impact-time distributions. This Direct PDF Propagation (DPP) method results in a multi-dimensional scattered dataset of the PDF of the state, which is highly challenging to process. No accurate results could be obtained, because of the structure of the DPP data and the high dimensionality. Therefore, the DPP method is less suitable for practical uncontrolled entry problems and the traditional MC method remains superior. Additionally, the MC method was used with two improved uncertainty models to obtain impact-time distributions, which were validated using observations of true impacts. For one of the two uncertainty models, statistically more valid impact-time distributions were obtained than in previous research.

Keywords: Uncertainty propagation; Statistical impact prediction; Space debris; Liouville equation

1. Introduction

Space debris presents the largest group of observable objects in Earth orbit since the in-orbit explosion of the US Transit-4A satellite in 1961. These objects have frequently entered the Earth's atmosphere of which some impacted the Earth's surface (Klinkrad, 2006). Considering the kinetic energy with which these bodies impact, it is valuable to have accurate predictions of the impact location and time.

Objects in orbital decay enter the Earth's atmosphere at the end of their life. During atmospheric entry, the body converts a large amount of kinetic energy into heat, which may result in breakup of the body. Impact prediction for these cases is performed using advanced tools that include breakup and survivability analysis (Portelli et al., 2004; Rochelle et al., 2004). Predictions of a decaying trajectory require a different approach.

The Space Surveillance Network (SSN) tracks a large number of objects orbiting the Earth. Orbit estimates resulting from these observations are published as Two-Line Elements (TLEs) in the Satellite Catalog (SATCAT). TLEs present a full description of the translational state of an object at the TLE epoch, as well as an estimate of its ballistic parameter. TLEs are updated in the SATCAT at

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regular intervals, resulting in a history of TLEs for each object. The orbit can be reconstructed by propagating the TLEs using the Simplified General Perturbations-4 (SGP4) simulator (Vallado et al., 2006). The SSN observations are also used to perform Tracking and Impact Predictions (TIPs)[4] for bodies that have a higher than 5% risk of reaching the surface without breaking up. These TIPs provide an estimated decay time and window, but do not provide any additional statistical information. Predictions of the decaying trajectory of an object are highly influenced by uncertainties in the initial state of the object and the models that are used for propagation. Therefore, statistical methods are required to perform impact predictions.

In the research of Ronse and Mooij (2014) a method is proposed to obtain statistical impact predictions using the Monte Carlo (MC) method. A 6 Degrees-of-Freedom (DoF) simulator was used to perform a large number of simulations with a randomly varying initial state and atmospheric density. The data were used to obtain a distribution of the impact time, which can be used to derive the distribution of the impact location using the ground track of the orbit. Accurate statistical predictions were obtained, which were validated with re-entry observations. A property of the Monte Carlo method is that the solution approaches the exact solution for an increasing number of simulations. However, a large number of simulations is required to obtain accurate results, which results in a large computational effort.

In recent research, methods are proposed that have the potential to provide more accurate results with the same or even less computational effort than MC. These methods directly propagate uncertainty, expressed by probability distributions, in time. In the research of Halder and Bhattacharya (2011), a method is presented that models the uncertainty in the state of an entry vehicle by a Probability Density Function (PDF) and propagates this PDF in time. The method uses a direct solution of the stochastic Liouville equation, which is a quasi-linear Partial Differential Equation (PDE) that encompasses the conservation of probability. Results are obtained for an entry vehicle and compared with MC, and it is argued that the method provides more accurate results with less computational effort. This method will be referred to as the Direct PDF Propagation (DPP) method.

In addition to the direct method, several approximate methods can be used to propagate uncertainty in time (Pantano and Shotorban, 2007; Prabhakar et al., 2010; Attar and Vedula, 2008; Terejanu et al., 2008). These methods, propagate a parametric model of the PDF of the state. Promising results were obtained with these methods and some are successfully applied to spaceflight related problems (Xu, 2009; Xu, 2011). Although these methods can provide significant reductions in computational effort, the DPP method potentially provides a higher accuracy.

In this paper, the DPP method is applied to statistical impact prediction, because of its promising accuracy. Statistical impact predictions are performed with the models developed by Ronse and Mooij (2014) of the Delta-K rocket body. Because 6-DoF trajectory simulations are performed, the dimension of this problem is considerably larger than the examples shown in Halder and Bhattacharya (2011). The goal is to investigate the limitations, accuracy and computational performance, and to compare the method to the MC method. Additionally, an improved uncertainty model is presented and used to obtain statistical impact predictions, which are validated and compared to the results of Ronse and Mooij (2014).

The paper starts with a discussion on the theory of the DPP method in Section 2. This theory is applied to develop simulators for statistical impact prediction, discussed in Section 3. An important part of the prediction method is the uncertainty model presented in Section 4, which determines the outcome of the predictions. The results of the research on the DPP method for statistical impact prediction are given in Section 5. The results of the statistical impact predictions with the improved uncertainty model are presented in Section 6. Finally, Section 7 describes the conclusions that are drawn from the research.

2. Probability density propagation

The trajectory of an object can be described using equations of motion that are defined as an initial-value problem of the form:

$$\dot{\boldsymbol{X}} = \boldsymbol{F}(\boldsymbol{X}(t), \boldsymbol{p}, t) \quad \text{with} \quad \boldsymbol{X}(0) = \boldsymbol{X_0} \tag{1}$$

where $\boldsymbol{X}_0$ is the initial state of the object. Solving these equations results in a unique solution of the trajectory. If uncertainty is present in the initial state, the trajectory cannot be defined in a deterministic way. The uncertainty in the initial state can be modeled by defining the initial state using a PDF, assigning a probability value to the possible initial states.

Instead of propagating a single trajectory using Eq. (1), the PDF of the state can be propagated in time. Hereby, all possible trajectories are propagated at once. The evolution of the PDF of the state, or the stochastic evolution of the state, is governed by the *Liouville Equation*[5] (Halder and Bhattacharya, 2011):

$$\frac{\partial \varphi(\boldsymbol{X}(t), t)}{\partial t} + \sum_{i=1}^{N_S} \frac{\partial}{\partial X_i} [\varphi(\boldsymbol{X}(t), t) F_i(\boldsymbol{X}(t), t)] = 0 \tag{2}$$

where $\varphi(\boldsymbol{X}(t), t)$ is the PDF of the state and N_S is the number of states in the state-derivative model, Eq. (1). This equation is a mathematical description of the conservation

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[4] The TLEs and TIPs are obtained from a catalog available at https://www.space-track.org/.

[5] The Liouville Equation is a special case of the Fokker-Planck Equation, which also includes a diffusion term (Gardiner, 2004).