Astroparticle Physics 103 (2018) 115–121

Contents lists available at ScienceDirect

Astroparticle Physics

journal homepage: www.elsevier.com/locate/astropartphys

The speed of gravitational waves and power-law solutions in a scalar-tensor model

L.N. Granda*, D.F. Jimenez

Departamento de Fisica, Universidad del Valle A.A. Cali 25360, Colombia

ARTICLE INFO

Article history:
Received 27 March 2018
Revised 27 July 2018
Accepted 5 August 2018
Available online 6 August 2018

ABSTRACT

One of the most relevant solutions in any cosmological model concerning the evolution of the universe is the power-law solution. For the scalar-tensor model of dark energy with kinetic and Gauss Bonnet couplings, it is shown that we can conserve the power-law solution and at the same time meet the recent observational bound on the speed of gravitational waves. In the FRW background the anomalous contribution to the speed of gravitational waves, coming from the kinetic and Gauss-Bonnet couplings, cancel each other for power-law solutions. It is shown that by simple restriction on the model parameters we can achieve a non-time-dependent cancellation of the defect in the velocity of the gravitational waves. The model can realize the cosmic expansion with contributions from the kinetic and Gauss-Bonnet couplings of the order of $\mathcal{O}(1)$ to the dark energy density parameter. The results are valid on the homogeneous FRW background and the limitations of the approach are discussed.

1. Introduction

The recent detection of gravitational waves (GW) from the merger of neutron stars in the system GW170817 together with its electromagnetic counterpart, the gamma-ray burst GRB 170817A [1–3], imposes strong bound on the speed of GW c_g which should differ from the speed of light c by at least one part in 10^{15}. At the same time this bound on the speed of GW translates into strong constraints on a widely studied scalar-tensor theories of gravity [4,5] which have successfully explained some of the aspects of the current dark energy problem (see reviews [6–10]). Horndeski theories are the most general scalar-field theories with equations of motion with no higher than second order derivatives. They include the known dark energy models of quintessence, k-essence, non-minimal coupling of the scalar field to curvature, to the Gauss-Bonnet invariant and non-minimal kinetic couplings to curvature, among other interactions. The interaction terms containing the coupling of the scalar field to the Gauss–Bonnet invariant, and the coupling between the scalar kinetic term and the curvature appear, among others, in the α'-expansion of the string effective action [11,12]. The non-minimal couplings of the kinetic term to curvature were considered in [13,14] to study inflationary attractor solutions, in [15] to find a connection with the cosmological constant, and a variety of solutions for the different cosmological epochs, particularly for late time acceleration, were found in [16–22]. The coupling between the scalar field and the Gauss-Bonnet invariant has been proposed to address the dark energy problem in [23], where it was found that quintessence or phantom phase may occur in the late time universe. Different aspects of accelerating cosmologies with GB correction have been also discussed in [24–28], and a modified GB theory applied to dark energy have been suggested in [29]. Late time cosmological solutions in a model that includes both, non-minimal kinetic coupling to curvature and coupling of the scalar field to the Gauss–Bonnet invariant, have been studied in [30–32].

A very important feature of these general scalar-tensor models is that they predict an anomalous GW speed ($c_g \neq c$), entering in contradiction with the observed results from GW170817 and GRB 170817A, and therefore are serious candidates to be discarded as dark energy models. The implications that this discovery has for the nature of dark energy (DE), apart form the tests of General Relativity, have been highlighted in [33–38]. Constraints imposed by the speed of GW on a scalar-tensor theories were analyzed in [37,39–42]. In [43] it was shown that the scalar field should be conformally coupled to the curvature in order to avoid the restrictions imposed by the speed of GW. Restrictions on vector-tensor theories have also been analyzed in [38,43]. The GW constraints on the coupling of the scalar kinetic term to the Einstein tensor and the scalar field to the GB invariant have been studied in [44], and restrictions on the beyond Horndeski parameters have been analyzed in [45,46]. According to all above studies, the overall conclu-

* Corresponding author.
E-mail addresses: luis.granda@correounivalle.edu.co (L.N. Granda), jimenez.diego@correounivalle.edu.co (D.F. Jimenez).

https://doi.org/10.1016/j.astropartphys.2018.08.001
0927-6505/

sion is that the constraint on α_g leaves only models that are conformally coupled to gravity, including models where the gravity is minimally coupled and models such as $f(R)$ gravity. A model independent phenomenological analysis of Horndeski theory on FRW background is given in [47], where it was shown that Horndeski theory with arbitrary G_4 and G_5 can hardly account for $c_g = c$ and explain the accelerating expansion without fine-tuning. Despite the severe restrictions imposed by GRB 170817A, in this work we show that there is an important solution, namely the power-law expansion in the frame of a scalar-tensor model with non-minimal kinetic and Gauss–Bonnet couplings, that passes the test of the speed of GW and remains consistent, at least in the FRW background, even after GRB 170817A. The relevance of the power-law solutions lies in the fact that they describe the different asymptotic regimes of expansion, depending on the type of matter that dominates throughout the evolution of the universe. We show that the cancelation between the anomalous contributions to c_g coming from the kinetic and GB couplings is not time-dependent and the model parameters don't require to be tuned beyond what is necessary to comply with the dark energy observations.

The limitation of the present approach lies in fact that we used $\dot{phi}$, $\ddot{\phi}$ and H, defined on the homogeneous FRW background, in the cancellation of the GW speed anomaly. In other words, the cancelation between the contributions from the interaction Lagrangians occurs at the cost of "tuning" the background manifold, while a definitive cancelation of the GW speed anomaly demands a covariant approach which makes it independent of the background metric. A discussion on the covariant approach is given. The paper is organized as follows. In section II we give the general equations expanded on the FRW metric, and find the power-law solutions for quintessence-like and phantom expansion. In Section 3 we give the speed of GW for the model on the homogeneous FRW background, and find the restrictions on the parameters that cancel the anomalous contribution to the speed of GW. In Section 4 we give a summary and discussion.

2. The model and power-law solutions

We consider the following string motivated action which includes the Gauss Bonnet coupling to the scalar field and kinetic couplings to curvature. These terms are present in the next to leading α' corrections in the string effective action (where the coupling coefficients are functions of the scalar field) [11,12].

$$S = \int d^4x\sqrt{-g}\Big[\frac{1}{16\pi G}R - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + F_1(\phi)G_{\mu\nu}\partial^\mu\phi\partial^\nu\phi - V(\phi) - F_2(\phi)\mathcal{G}\Big] \quad (1)$$

where $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R$, $\mathcal{G}$ is the 4-dimensional GB invariant $\mathcal{G} = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$. The coupling $F_1(\phi)$ has dimension of $(length)^2$, and the coupling $F_2(\phi)$ is dimensionless. Note that we are not considering derivative terms that are not directly coupled to curvature, of the form $\Box\phi\partial_\mu\phi\partial^\mu\phi$ and $(\partial_\mu\phi\partial^\mu\phi)^2$. The equations derived from this action contain only second derivatives of the metric and the scalar field.

In the spatially-flat Friedmann–Robertson–Walker (FRW) metric,

$$ds^2 = -dt^2 + a(t)^2\big(dr^2 + r^2 d\Omega^2\big), \quad (2)$$

where $a(t)$ is the scale factor, the set of equations describing the dynamical evolution of the FRW background and the scalar field for the model (1) are ($8\pi G = \kappa^2 = M_p^{-2}$)

$$H^2 = \frac{\kappa^2}{3}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi) + 9H^2F_1(\phi)\dot{\phi}^2 + 24H^3\frac{dF_2}{d\phi}\dot{\phi}\right) \quad (3)$$

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} + 3H^2\left(2F_1(\phi)\ddot{\phi} + \frac{dF_1}{d\phi}\dot{\phi}^2\right) + 18H^3F_1(\phi)\dot{\phi} + 12H\dot{H}F_1(\phi)\dot{\phi} + 24\big(\dot{H}H^2 + H^4\big)\frac{dF_2}{d\phi} = 0 \quad (4)$$

Here we will consider the string inspired model with exponential couplings, and additionally, we consider an exponential potential given by

$$F_1(\phi) = \xi e^{\alpha\kappa\phi/\sqrt{2}}, \quad F_2(\phi) = \eta e^{\alpha\kappa\phi/\sqrt{2}}, \quad V(\phi) = V_0 e^{-\alpha\kappa\phi/\sqrt{2}} \quad (5)$$

Where the coupling η may be related to the string coupling g_s as $\eta \sim 1/g_s^2$. The de Sitter solution for the model (1) with couplings and potential given by (5) follows from Eqs. (3) and (4) by setting $H = const. = H_c$ and $\phi = const. = \phi_c$, which gives

$$H_0^2 = \frac{M_p^2}{8\eta}e^{-2\alpha\kappa\phi_c/\sqrt{2}} \quad (6)$$

This model admits the power-law solution for quintessence-like expansion [48,49]

$$H = \frac{p}{t}, \quad \phi = \phi_0 \ln\frac{t}{t_1}, \quad (7)$$

and the solution

$$H = \frac{p}{t_s - t}, \quad \phi = \phi_0 \ln\frac{t_s - t}{t_1} \quad (8)$$

for phantom power-law expansion. These solutions lead to the effective equation of state w_{eff}

$$w_{eff} = -1 \pm \frac{2}{3p} \quad (9)$$

where the lower sign is for phantom solution. The equations (7) and (8), after being replaced in the Friedmann equation (3), lead to the following restrictions

$$\frac{3p^2}{\kappa^2} = \frac{1}{2}\phi_0^2 + V_0t_1^2 + \frac{9\xi p^2}{t_1^2}\phi_0^2 \pm \frac{48\eta p^3}{t_1^2} \quad (10)$$

where we fixed ϕ_0 according to

$$\frac{2\sqrt{2}}{\alpha\kappa} = \phi_0, \quad (11)$$

in order to get the power t^{-2} from the interacting terms. The equation of motion (4) gives

$$(\pm 3p - 1)\phi_0^2 - 2V_0t_1^2 + \frac{6\xi p^2(\pm 3p - 2)}{t_1^2}\phi_0^2 \pm \frac{48\eta p^3(\pm p - 1)}{t_1^2} = 0 \quad (12)$$

where the lower minus sign follows for the phantom solution. The stability properties of the solutions (7) and (8) has been performed in [48] for the case of $V_0 = 0$ and in [49] for more general cases. As will be shown below, the restrictions (10)–(12) with an additional constraint coming from the velocity of the GW can be solved consistently.

3. Restriction from the speed of gravitational waves

The generalized Galileons, which are equivalent to Horndeski theory in four dimensions, represent the most general scalar field theories having second-order field equations and are described by the Lagrangian density [5,50]

$$\mathcal{L} = \sum_{i=2}^{5}\mathcal{L}_i \quad (13)$$