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Spectral features in the cosmic ray fluxes

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ABSTRACT

The cosmic ray energy distributions contain spectral features, that is narrow energy regions where the slope of the spectrum changes rapidly. The identification and study of these features is of great importance to understand the astrophysical mechanisms of acceleration and propagation that form the spectra. In first approximation a spectral feature is often described as a discontinuous change in slope, however very valuable information is also contained in its width, that is the length of the energy interval where the change in spectral index develops. In this work we discuss the best way to define and parameterize the width a spectral feature, and for illustration discuss some of the most prominent known structures.

1. Introduction

The spectra of cosmic rays (CR) extend to a very broad energy range with a smooth shape that, for energy $E \gtrsim 30$ GeV, is usually described as an ensemble of adjacent energy intervals, where the energy distribution is a simple power law ($\phi(E) \simeq K\, E^{-\alpha}$), separated by "spectral features", that is narrow regions where the slope (or spectral index) of the flux undergoes a rapid change. The features can be softenings or hardenings of the spectrum, and appear as "knee–like" or "ankle–like" in the usual log–log graphic representation of the spectrum. Prominent and well known examples of features in the all particle spectrum are in fact the "Knee" at $E \simeq 3$ PeV, and the "Ankle" at $E \simeq 4$ EeV.

The simple description outlined above is an approximation, because it is likely that the CR spectra are not, even in a limited range of energy, exactly of power law form, and the spectral indices are always slowly evolving with energy; however the identification and study of discrete spectral features can be considered as a natural and useful task.

It is obviously very desirable, and in fact ultimately necessary, to describe the CR spectral features in the framework of astrophysically motivated models, and in terms of parameters that have a real physical meaning, and in the literature there are several alternative models to interpret the observations. On the other hand, it is useful to have a purely phenomenological description of the shape of the spectral features, as an intermediate step that can be used as a guide in the construction of astrophysical models.

In first order approximation, a spectral feature can be described as infinitely narrow, with the spectral index that changes discontinuously. In this limit a feature it is completely described by four parameters: E_b the break energy, that gives its position, α_1 and α_2 the spectral slopes before and after the break, and the absolute normalization of the flux.

It is obvious that the hypothesis of a discontinuous change in spectral slope is unphysical, and this suggests that a phenomenological description of a spectral feature should include at least one additional parameter. A simple and convenient parameterization of the spectral shape of the CR all particle spectrum in the region of the Knee has been introduced by Ter–Antonyan and Haroyan [1] and later adopted by Schatz [2] and also used by the HESS collaboration [3] to describe the spectrum of electrons plus positrons. This parameterization can be applied to the description of both softening and hardening spectral features and (with E_0 is an arbitrary reference energy) has the form:

$$\phi(E) = K_0 \left(\frac{E}{E_0}\right)^{-\alpha_1} \left[1 + \left(\frac{E}{E_b}\right)^{\frac{1}{w}}\right]^{-(\alpha_2-\alpha_1)\,w} \qquad (1)$$

that contains one additional parameter, the width $w > 0$ (note that the authors of [1,2] use the parameter $\varepsilon = 1/w$). It can be observed that the two sets of parameters $\{K, E_b, \alpha, \alpha', w\}$ and $\{K', E_b, \alpha', \alpha, -w\}$ (with $K' = K\,(E_0/E_b)^{(\alpha-\alpha')}$) generate identical curves. Imposing the constraint $w > 0$ eliminates this ambiguity, selecting the solution where the parameters $\alpha_{1,\,2}$ are equal to the asymptotic spectral indices for low and high energy.

Some examples of the spectral shapes of this parameterization are shown in Fig. 1. For a more precise understanding of the "geometrical meaning" of w it is useful to consider the energy dependence of the spectral index of a flux described by Eq. (1):

$$\alpha(E) \equiv -\frac{d\ln\phi}{d\ln E} = \overline{\alpha} + \frac{\Delta\alpha}{2}\,\tanh\left[\frac{\ln(E/E_b)}{2w}\right]. \qquad (2)$$

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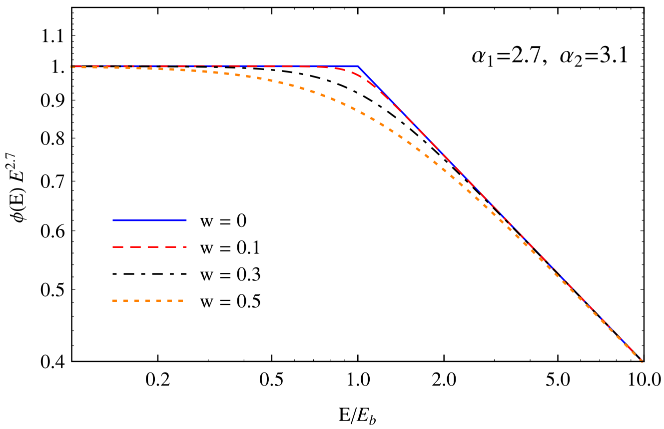

Fig. 1. Example of a (softening) spectral feature described by the parameterization of Eq. (1). The spectral indices before and after the break are $\alpha_1 = 2.7$ and $\alpha_2 = 3.1$. The different curves are calculated as the limit for $w \to 0$, and with $w = 0.1$, 0.3 and 0.5.

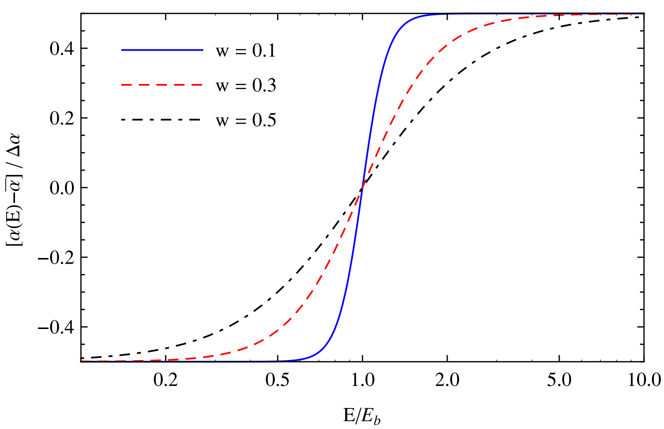

Fig. 2. Energy dependence of the spectral index (see Eq. (2)). The three curves correspond to three values of the width parameter ($w = 0.1$, 0.3 and 0.5).

In this equation $\overline{\alpha} = (\alpha_2 + \alpha_1)/2$ is the average of the two spectral indices before and after the break, and $\Delta\alpha = (\alpha_2 - \alpha_1)$ is the total change in spectral index across the break (some numerical examples are shown in Fig. 2). It is straightforward to see that w gives the width of interval in $\log E$ where the step in spectral index develops.

The limit for $w \to 0$ of Eq. (1) is a broken power law with spectral index α_1 for $E < E_b$, and α_2 for $E > E_b$, and the same limit for Eq. (2) yields:

$$\lim_{w\to 0} \alpha(E) = \begin{cases} \overline{\alpha} - \frac{\Delta\alpha}{2} = \alpha_1 & \text{for } E < E_b \\ \overline{\alpha} + \frac{\Delta\alpha}{2} = \alpha_2 & \text{for } E > E_b \end{cases} \tag{3}$$

and corresponds to a discontinuous jump of the spectral index. More in general, one has that the asymptotic values (for $E \to 0$ and $E \to \infty$) of the spectral index are α_1 and α_2, and at the break energy E_b the spectral index takes the average value: $\alpha(E_b) = \overline{\alpha}$. The step in spectral index $\Delta\alpha$ develops symmetrically in $\log E$, and the energies $E_{f_\pm}$ where the spectral index takes the values:

$$\alpha(E_{f_\pm}) = \overline{\alpha} \pm \frac{\Delta\alpha}{2}\, f \tag{4}$$

(with $0 \le f < 1$) are given by:

$$\log E_{f_\pm} = \log E_b \pm w \, \log \left[\frac{1+f}{1-f}\right], \tag{5}$$

so that the two values $\log E_{f_\pm}$ are placed symmetrically with respect to $\log E_b$. The total range of $\log E$ (centered on $\log E_b$) where the spectral index varies by $\Delta\alpha/2$ is then:

$$(\Delta \log_{10} E)_{\Delta\alpha/2} = (\log_{10} 9)\, w \simeq 0.954\, w. \tag{6}$$

This allows to attribute a simple and easy to remember physical meaning to w . The value $w \simeq 1$ corresponds to a spectral feature that develops in approximately a decade of energy, and a feature of width $w \simeq 0.1$ has an energy extension that is approximately a factor $\approx 10^{0.1} \simeq 1.25$.

The width w is also related to the derivative of the spectral index at the break energy by the simple relation:

$$\left.\frac{d\alpha(E)}{d\ln E}\right|_{E=E_b} = \frac{\Delta\alpha}{4\,w}\,. \tag{7}$$

Recently the AMS02 collaboration has presented fits to the rigidity spectra of the proton an helium spectra [12,13] using the functional form (expressed here as a function of energy):

$$\phi(E) = K \left(\frac{E}{E_0}\right)^{-\alpha_1} \left[1 + \left(\frac{E}{E_b}\right)^{-(\alpha_2-\alpha_1)/s}\right]^s. \tag{8}$$

Eqs. (1) and (8) are in fact different parameterizations of the same ensemble of curves. The parameter s used in Eq. (8) is related to the width w of Eq. (1) by:

$$s = -(\alpha_2 - \alpha_1)\, w \tag{9}$$

and therefore Eqs. (1) and (8) are equivalent. The parameterization used by the AMS02 collaboration suffers from the same ambiguity present for the form of Eq. (1), because (with an appropriate modification of the normalization factor) the two sets of parameters $\{\alpha, \alpha', s\}$ and $\{\alpha', \alpha, -s\}$ correspond to identical curves. The choice of the set where the quantities $\alpha_{1,2}$ are the asymptotic spectral indices of the curve for low and high energy, corresponds to the set of parameters with $s > 0$ if the spectral feature is a hardening, and the set with $s < 0$ if the spectral feature is a softening.

Even if the two parameterizations of Eqs. (1) and (8) are mathematically equivalent, we find that the use of the width parameter w is preferable because of its more transparent and intuitive physical meaning. In addition, when performing fits to data, the quantities in the pair $\{s, \Delta\alpha\}$ are in general much more strongly correlated than the quantities in the pair $\{w, \Delta\alpha\}$.

As discussed above, the spectral index of a flux described by Eq. (1) or (8) is symmetric in $\log E$. It is potentially interesting to have a more flexible functional form to describe a spectral feature that allows for the possibility that the spectral index changes more rapidly before or after the break energy. A simple generalization of Eq. (1) that depends on one more parameter, can be obtained, keeping for E_b the same definition, that is the energy where the spectral index takes the average value:

$$\alpha(E_b) = \frac{(\alpha_1 + \alpha_2)}{2} \tag{10}$$

and introducing two different widths to the left and right of the break energy. This results in the form:

$$\phi(E) = \begin{cases} K_0 \left(\frac{E}{E_0}\right)^{-\alpha_1} \left[1 + \left(\frac{E}{E_b}\right)^{\frac{1}{w_L}}\right]^{-\Delta\alpha\, w_L} & \text{for } E < E_b \\ K_0\, 2^{\Delta\alpha\,(w_R - w_L)} \left(\frac{E}{E_0}\right)^{-\alpha_1} \left[1 + \left(\frac{E}{E_b}\right)^{\frac{1}{w_R}}\right]^{-\Delta\alpha\, w_R} & \text{for } E > E_b\,, \end{cases} \tag{11}$$

so that the spectral index $\alpha(E)$ takes the form:

$$\alpha(E) = \begin{cases} \overline{\alpha} + \frac{\Delta\alpha}{2} \, \tanh\left[\frac{\ln(E/E_b)}{2w_L}\right] & \text{for } E < E_b \\ \overline{\alpha} + \frac{\Delta\alpha}{2} \, \tanh\left[\frac{\ln(E/E_b)}{2w_R}\right] & \text{for } E > E_b\,. \end{cases} \tag{12}$$

For this parameterization the flux and its first derivative (i.e. the spectral index) are continuous, but the second derivative is discontinuous at the point $E = E_b$. Taking the derivative of the spectral