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The assessment of the semi-analytical method in the long-term orbit prediction of Earth satellites ⋆

WANG Da-wei[1,2△] TANG Jing-shi[1,2△△] LIU Lin [1,2]
MA Chao-wei[3] HAO Shi-feng[3]

(1 School of Astronomy and Space Science, Nanjing University Nanjing 210023)
(2 Institute of Space Environment and Astrodynamics, Nanjing University Nanjing 210023)
(3 Beijing Institute of Tracking and Telecommunications Technology Beijing 100094)

Abstract To understand the long-term evolution and distribution of the space objects, it is necessary to predict their orbits. Compared with the short-term prediction of a few days, the priority concerns of the long-term orbit prediction are the calculation speed, and the accuracies of major orbital elements, including the semi-major axis and eccentricity which define the shape of the orbit, as well as the orbital inclination and the right ascension of ascending node which define the orientation of the orbit. Given these requirements, it is preferable to adopt the semi-analytical method, which averages the system over the orbital period, and integrates the averaged system using the numerical method. It is not new, however, in the available literature, we can hardly find a quantitative assessment regarding its accuracy and speed when it is applied to various types of orbits. In this paper, we would like to report our implementation and assessment of the semi-analytical method, expecting that it would help to estimate its feasibility in the long-term orbit prediction. The quantitative assessment covers the commonly used orbits for the Earth satellites. In some rare and special cases where the performance of our method appears abnormal, we discuss the reasons and possible solutions. We hope our results can provide some useful reference for the similar applications of the semi-analytical method since our method is a relatively common approach in terms of both accuracy and implementation.

Key words celestial mechanics, planets and satellites: dynamical evolution, methods: analytical

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△sj_cbl@163.com

△△jstang@nju.edu.cn

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1. INTRODUCTION

Semi-analytical method is a common method for long-term orbit prediction and is widely used in the solar system dynamics (e.g. Widsom 1982[1]), where a certain numerical method is used to integrate or map the averaged system. Because the perturbing acceleration is averaged, for example over an orbital period, it may have a larger integration step size and faster speed compared with the numerical method. It will be also more accurate than the pure analytical method owing to the numerical propagation of the averaged system.

The long-term orbit evolution and distribution of space objects is an important subject in the space object surveillance, which could involve tens of thousands of objects. By 17th February 2016, there have already been 15431 objects in the Two-line Element (TLE) catalogue. The priority concern for the long-term prediction is the accuracy of major orbital elements, including the semi-major axis a and eccentricity e which define the shape of the orbit, and the orbital inclination i and the right ascension of ascending node Ω which define the orientation of the orbit, while the exact position (the anomaly M) is not so important. Therefore, it is always preferable to use the semi-analytical method to obtain quickly and accurately the orbital shapes and orientations of these objects over decades or even a century.

As for the semi-analytical method itself, one of the key points is how to obtain the averaged perturbing acceleration. Normally, the main approach is to formulate the averaged accelerations analytically, which is called the analytical average in this paper. The system can be studied in terms of the non-Hamiltonian and Hamiltonian mechanics respectively. Liu (2000)[2] used the perturbation method (Kozai 1959[3]) to obtain a series of perturbation solutions of the Keplerian elements. In the middle of 70s, Paul Cefola and his colleagues (Cefola 1972[4], 1976[5]) proposed the Draper Semi-analytical Satellite Theory (DSST) based on the equinoctial elements. And it has been applied to an orbit propagator called DSST Standalone (Cefola et al. 2014[6]). Danielson et al. (1995)[7] presented an excellent summary on the mathematical technique of DSST. As for the Hamiltonian approach, people tend to adopt the canonical Lie transformation theory (Deprit 1969[8]) to eliminate the short-period terms and obtain the averaged perturbing acceleration (Metris & Exertier 1995[9], Lara et al. 2014a[10], Lara et al. 2014b[11], Valk et al. 2009[12], Deleflie et al. 2005[13], Deleflie et al. 2011[14], Bruinsma et al. 1997[15]). Some others directly average the Hamiltonian function over the orbital period without the use of canonical transformation (Valk et al. 2008[16], Valk & Lemaitre 2008[17]). Besides, another method, referred to as the numerical average, is also mentioned in some literature (e.g. Bruinsma et al. 1997[15], Klinkrad 2006[18], Wu 2012[38], Gao 2008[39]), which numerically computes the quadrature of the perturbing acceleration over one orbital period and regards it as the averaged acceleration.

Although there are many reports on the various implementations or applications of the semi-analytical method, it is rarely seen that the semi-analytical methods are quantitatively assessed regarding the accuracy and speed in the general sense, especially for the application to the long-term prediction of various types of Earth satellite orbits. The purpose of this