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Chaotic dynamics in the (47171) Lempo triple system

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ABSTRACT

We investigate the dynamics of the (47171) Lempo triple system, also known by 1999 TC_{36}. We derive a full 3D *N*-body model that takes into account the orbital and spin evolution of all bodies, which are assumed triaxial ellipsoids. We show that, for reasonable values of the shapes and rotational periods, the present best fitted orbital solution for the Lempo system is chaotic and unstable in short time-scales. The formation mechanism of this system is unknown, but the orbits can be stabilised when tidal dissipation is taken into account. The dynamics of the Lempo system is very rich, but depends on many parameters that are presently unknown. A better understanding of this systems thus requires more observations, which also need to be fitted with a complete model like the one presented here.

1. Introduction

A non-negligible fraction of the small bodies in the solar system are in multiple systems, mostly composed by binaries (e.g. Noll et al., 2008). The shapes of these small objects are usually irregular (Lacerda and Jewitt, 2007), resulting in important asymmetries in the gravitational potential. The dynamics of these objects is thus very rich, as these asymmetries lead to strong spin-orbit coupling, where the rotation rate can be captured in a half-integer commensurability with the mean motion (Colombo, 1965; Goldreich and Peale, 1966). For very eccentric orbits or large axial asymmetries, the rotational libration width of the individual resonances may overlap, and the dynamics becomes chaotic (Wisdom et al., 1984; Wisdom, 1987). When a third body is added to the problem, the mutual gravitational perturbations also introduce additional spin-orbit resonances at the perturbing frequency (Goldreich and Peale, 1967; Correia et al., 2015; Delisle et al., 2017).

The spin and orbital dynamics of small-body binaries has been object of many previous studies. However, due to the complexity of the spin-orbit interactions, in general these works either focus on the spin or in the orbital dynamics, i.e., they study the spin of a triaxial body around a distant companion (e.g. Batygin and Morbidelli, 2015; Naidu and Margot, 2015; Jafari Nadoushan and Assadian, 2016), or the motion of a test particle around a triaxial body (e.g. Mysen and Aksnes, 2007; Scheeres, 2012; Lages et al., 2017). Moreover, for simplicity, most studies consider that the spin axis is always normal to the orbital plane, and when a third body is considered, the orbits are also made coplanar.

(47171) Lempo (also known by 1999 TC_{36}) is a triple system. It is classified as a *Plutino*, since it is in a 3/2 mean-motion resonance with Neptune, like Pluto. The primary was discovered in 1999 at the Kitt Peak Observatory (Rubenstein and Strolger, 1999). A similar size secondary was identified in 2001 from images obtained by the Hubble Space Telescope (Trujillo and Brown, 2002). Subsequent observations lead to the determination of the orbit of the secondary with a period of about 50 days (Margot et al., 2005). A third component, also of similar size, was finally discovered in 2007 also using observations from the Hubble Space Telescope (Jacobson and Margot, 2007). The third body is actually much closer to the primary than the secondary, with an orbital period of only 1.9 days (Benecchi et al., 2010). The Lempo system can thus be characterised as an inner close binary with an outer circumbinary companion, with all three components being of identical sizes, which is unique.

The name *Lempo* actually refers to the larger component of the inner binary, while the smaller component is named *Hiisi*, and the outer circumbinary component is named *Paha*. The best fitted orbits for the Lempo system are eccentric (~ 0.1 for the inner orbit and ~ 0.3 for the outer one) and present a mutual inclination of about 10 degrees (Benecchi et al., 2010). The three bodies have diameter sizes within $100 - 300$ km (Mommert et al., 2012), which is consistent with a large triaxiality. Therefore, since the two inner components are very close to each other, we expect to observe a strong spin-orbit coupling in this system.

In this paper we derive a full 3D model (for the orbits and spins) that is suitable to describe the motion of a $N-$body system, where all bodies are assumed triaxial ellipsoids (Section 2). This model is able to simultaneously handle spin and orbital dynam-

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ics without any kind of restrictions. We then apply our model to the Lempo system in Section 3, and show that the present best fitted solution corresponds to a chaotic system for reasonable values of the unknown triaxiality. In Section 4 we analyse the impact of tidal evolution on the final evolution of the system. Finally, in last section we discuss our results.

2. Model

In this section we derive a very general model that is suited to study a system of N-bodies with ellipsoidal shapes. Our model is valid in 3D for the orbits and individual spins. We make no particular assumption on the spin axes. We use cartesian inertial coordinates, and quaternions to deal with the rotations.

2.1. Potential of an ellipsoidal body

We consider an ellipsoidal body of mass m, and chose as reference the cartesian inertial frame ($\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k}$). In this frame, the rotational angular velocity and angular momentum vectors of the body are given by $\boldsymbol{\omega} = (\omega_i, \omega_j, \omega_k)$ and $\boldsymbol{L} = (L_i, L_j, L_k)$, respectively, which are related through the inertia tensor $\mathcal{I}$ as

$$\boldsymbol{L} = \mathcal{I} \cdot \boldsymbol{\omega} \quad \Leftrightarrow \quad \boldsymbol{\omega} = \mathcal{I}^{-1} \cdot \boldsymbol{L} \, , \tag{1}$$

where

$$\mathcal{I} = \begin{bmatrix} I_{11} & I_{12} & I_{13} \\ I_{12} & I_{22} & I_{23} \\ I_{13} & I_{23} & I_{33} \end{bmatrix} . \tag{2}$$

The gravitational potential of the ellipsoidal body at a generic position $\boldsymbol{r}$ from its center-of-mass is given by (e.g., Goldstein, 1950)

$$V(\boldsymbol{r}) = -\frac{Gm}{r} + \frac{3G}{2r^3}\left[\hat{\boldsymbol{r}} \cdot \mathcal{I} \cdot \hat{\boldsymbol{r}} - \frac{1}{3}\mathrm{tr}(\mathcal{I})\right] , \tag{3}$$

where G is the gravitational constant, $\hat{\boldsymbol{r}} = \boldsymbol{r}/r = (\hat{x}, \hat{y}, \hat{z})$ is the unit vector, and $\mathrm{tr}(\mathcal{I}) = I_{11} + I_{22} + I_{33}$. We neglect terms in $(R/r)^3$, where R is the mean radius of the body (quadrupolar approximation). Adopting the Lagrange polynomial $P_2(x) = (3x^2 - 1)/2$, we can rewrite the previous potential as

$$V(\boldsymbol{r}) = -\frac{Gm}{r} + \frac{G}{r^3}\Big[\big(I_{22} - I_{11}\big)P_2(\hat{y}) + \big(I_{33} - I_{11}\big)P_2(\hat{z}) + 3\big(I_{12}\hat{x}\hat{y} + I_{13}\hat{x}\hat{z} + I_{23}\hat{y}\hat{z}\big)\Big]. \tag{4}$$

2.2. Point-mass problem

We now consider that the ellipsoidal body orbits a point-mass M located at $\boldsymbol{r}$. The force between the two bodies is easily obtained from the potential energy of the system $U(\boldsymbol{r}) = MV(\boldsymbol{r})$ (Eq. (4)) as

$$\boldsymbol{F} = -\nabla U(\boldsymbol{r}) = \boldsymbol{f}(M, m, \boldsymbol{r}) + \boldsymbol{g}(M, \mathcal{I}, \boldsymbol{r}) + \boldsymbol{h}(M, \mathcal{I}, \boldsymbol{r}) \, , \tag{5}$$

with

$$\boldsymbol{f}(M, m, \boldsymbol{r}) = -\frac{GMm}{r^3}\boldsymbol{r} \, , \tag{6}$$

$$\boldsymbol{g}(M, \mathcal{I}, \boldsymbol{r}) = \frac{15\ GM}{r^5}\left[\frac{I_{22} - I_{11}}{2}\left(\hat{y}^2 - \frac{1}{5}\right) + \frac{I_{33} - I_{11}}{2}\left(\hat{z}^2 - \frac{1}{5}\right) + I_{12}\hat{x}\hat{y} + I_{13}\hat{x}\hat{z} + I_{23}\hat{y}\hat{z}\right]\boldsymbol{r}, \tag{7}$$

$$\boldsymbol{h}(M, \mathcal{I}, \boldsymbol{r}) = -\frac{3\ GM}{r^4}\Big[\big(I_{22} - I_{11}\big)\hat{y}\boldsymbol{j} + \big(I_{33} - I_{11}\big)\hat{z}\boldsymbol{k} + I_{12}(\hat{x}\boldsymbol{j} + \hat{y}\boldsymbol{i}) + I_{13}(\hat{x}\boldsymbol{k} + \hat{z}\boldsymbol{i}) + I_{23}(\hat{y}\boldsymbol{k} + \hat{z}\boldsymbol{j})\Big]. \tag{8}$$

We thus obtain for the orbital evolution of the system

$$\ddot{\boldsymbol{r}} = \boldsymbol{F}/\beta \, , \tag{9}$$

where $\beta = Mm/(M+m)$ is the reduced mass. The spin evolution of the ellipsoidal body can also be obtained from the force, by computing the gravitational torque. In the inertial frame we have:

$$\dot{\boldsymbol{L}} = \boldsymbol{T}(M, \mathcal{I}, \boldsymbol{r}) = -\boldsymbol{r} \times \boldsymbol{F} = -\boldsymbol{r} \times \boldsymbol{h} \, , \tag{10}$$

that is,

$$\boldsymbol{T}(M, \mathcal{I}, \boldsymbol{r}) = \frac{3\ GM}{r^3}\hat{\boldsymbol{r}} \times \Big[\big(I_{22} - I_{11}\big)\hat{y}\boldsymbol{j} + \big(I_{33} - I_{11}\big)\hat{z}\boldsymbol{k} + I_{12}(\hat{x}\boldsymbol{j} + \hat{y}\boldsymbol{i}) + I_{13}(\hat{x}\boldsymbol{k} + \hat{z}\boldsymbol{i}) + I_{23}(\hat{y}\boldsymbol{k} + \hat{z}\boldsymbol{j})\Big], \tag{11}$$

or

$$\boldsymbol{T} = \frac{3\ GM}{r^3}\begin{bmatrix} \big(I_{33} - I_{22}\big)\hat{y}\hat{z} - I_{12}\hat{x}\hat{z} + I_{13}\hat{x}\hat{y} + I_{23}(\hat{y}^2 - \hat{z}^2) \\ \big(I_{11} - I_{33}\big)\hat{x}\hat{z} + I_{12}\hat{y}\hat{z} + I_{13}(\hat{z}^2 - \hat{x}^2) - I_{23}\hat{x}\hat{y} \\ \big(I_{22} - I_{11}\big)\hat{x}\hat{y} + I_{12}(\hat{x}^2 - \hat{y}^2) - I_{13}\hat{y}\hat{z} + I_{23}\hat{x}\hat{z} \end{bmatrix} . \tag{12}$$

Apart from a sphere, in the inertial frame ($\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k}$) the inertia tensor (2) is not constant. We let $\mathcal{S}$ be the rotation matrix that allow us to convert any vector $\boldsymbol{u}_B$ in a frame attached to the body into the cartesian inertial frame $\boldsymbol{u}_I$, such that $\boldsymbol{u}_I = \mathcal{S}\,\boldsymbol{u}_B$. Thus, we have

$$\mathcal{I} = \mathcal{S}\,\mathcal{I}_B\mathcal{S}^T \quad , \quad \text{and} \quad \mathcal{I}^{-1} = \mathcal{S}\,\mathcal{I}_B^{-1}\mathcal{S}^T \, , \tag{13}$$

where $\mathcal{I}_B$ is the inertia tensor in the body frame. For principal axis of inertia $\mathcal{I}_B = \mathrm{diag}(A, B, C)$ and $\mathcal{I}_B^{-1} = \mathrm{diag}(A^{-1}, B^{-1}, C^{-1})$. The evolution of $\mathcal{S}$ over time is given by

$$\dot{\mathcal{S}} = \tilde{\boldsymbol{\omega}}\,\mathcal{S} \quad , \quad \text{and} \quad \dot{\mathcal{S}}^T = -\mathcal{S}^T\tilde{\boldsymbol{\omega}} \, , \tag{14}$$

with

$$\tilde{\boldsymbol{\omega}} = \begin{bmatrix} 0 & -\omega_k & \omega_j \\ \omega_k & 0 & -\omega_i \\ -\omega_j & \omega_i & 0 \end{bmatrix} . \tag{15}$$

In order to simplify the evolution of $\mathcal{S}$, a set of generalized coordinates to specify the orientation of the two frames can be used. Euler angles are a common choice, but they introduce some singularities. Therefore, here we use quaternions (eg. Kosenko, 1998). We denote $\boldsymbol{q} = (q_0, q_1, q_2, q_3)$ the quaternion that represents the rotation from the body frame to the inertial frame. Then

$$\mathcal{S} = \begin{bmatrix} q_0^2 + q_1^2 - q_2^2 - q_3^2 & 2(q_1q_2 - q_0q_3) & 2(q_1q_3 + q_0q_2) \\ 2(q_1q_2 + q_0q_3) & q_0^2 - q_1^2 + q_2^2 - q_3^2 & 2(q_2q_3 - q_0q_1) \\ 2(q_1q_3 - q_0q_2) & 2(q_2q_3 + q_0q_1) & q_0^2 - q_1^2 - q_2^2 + q_3^2 \end{bmatrix} , \tag{16}$$

and

$$\dot{\boldsymbol{q}} = \frac{1}{2}(0, \boldsymbol{\omega}) \cdot \boldsymbol{q} = \frac{1}{2}\begin{bmatrix} -\omega_iq_1 - \omega_jq_2 - \omega_kq_3 \\ \omega_iq_0 + \omega_jq_3 - \omega_kq_2 \\ -\omega_iq_3 + \omega_jq_0 + \omega_kq_1 \\ \omega_iq_2 - \omega_jq_1 + \omega_kq_0 \end{bmatrix} . \tag{17}$$

To solve the spin-orbit motion, we need to integrate equations (9), (10) and (17), using the relations (1), (13) and (16).

2.3. Two-body problem

Consider now that two ellipsoidal bodies with masses m_0 and m_1, and inertia tensors $\mathcal{I}_0$ and $\mathcal{I}_1$, respectively, orbit around each other at a distance $\boldsymbol{r}$ from their centers-of-mass. The total potential energy can be written from expression (3) as

$$U = -\frac{Gm_0m_1}{r} + \frac{3G}{2r^3}\left[\hat{\boldsymbol{r}} \cdot \mathcal{J} \cdot \hat{\boldsymbol{r}} - \frac{1}{3}\mathrm{tr}(\mathcal{J})\right] , \tag{18}$$

with $\mathcal{J} = m_0\mathcal{I}_1 + m_1\mathcal{I}_0$. This potential is very similar to the previous point-mass problem and the equations of motion are simply

$$\ddot{\boldsymbol{r}} = \boldsymbol{F}_{01}(\boldsymbol{r})/\beta_{01} \, , \tag{19}$$