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Internal gravity, self-energy, and disruption of comets and asteroids

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ABSTRACT

The internal gravity and self-gravitational energy of a comet, asteroid, or small moon have applications to their geophysics, including their formation, evolution, cratering, and disruption, the stresses and strains inside such objects, sample return, eventual asteroid mining, and planetary defense strategies for potentially hazardous objects. This paper describes the relation of an object's self-energy to its collisional disruption energy, and shows how to determine an object's self-energy from its internal gravitational potential.

Any solid object can be approximated to any desired accuracy by a polyhedron of sufficient complexity. An analytic formula is known for the gravitational potential of any homogeneous polyhedron, but it is widely believed that this formula applies only on the surface or outside of the object. Here we show instead that this formula applies equally well inside the object.

We have used these formulae to develop a numerical code which evaluates the self-energy of any homogeneous polyhedron, along with the gravitational potential and attraction both inside and outside of the object, as well as the slope of its surface. Then we use our code to find the internal, external, and surface gravitational fields of the Platonic solids, asteroid (216) Kleopatra, and comet 67P/Churyumov–Gerasimenko, as well as their surface slopes and their self-gravitational energies. We also present simple spherical, ellipsoidal, cuboidal, and duplex models of Kleopatra and comet 67P, and show how to generalize our methods to inhomogeneous objects and magnetic fields.

At present, only the self-energies of spheres, ellipsoids, and cuboids (boxes) are known analytically (or semi-analytically). The Supplementary Material contours the central potential and self-energy of homogeneous ellipsoids and cuboids of all aspect ratios, and also analytically the self-gravitational energy of a "duplex" consisting of two coupled spheres. The duplex is a good model for "contact binary" comets and asteroids; in fact, most comets seem to be bilobate, and might be described better as "dirty snowmen" than as "dirty snowballs".

1. Introduction

The internal gravity and self-gravitational energy of a mass distribution have applications to various problems in our Solar system, as well as to galactic and stellar dynamics. In the present context, the main applications are to the geophysics of solid objects such as comets, asteroids, planetesimals, small moons, *etc.*, including their formation, evolution, cratering, and disruption (whether collisional, rotational, or tidal); the stresses and strains inside such objects (whether modeled analytically, as by Dobrovolskis (1990), semi-analytically, as by Dobrovolskis (1982), or numerically with Finite Element Modeling, as by Hirabayashi and Scheeres (2014); sample return; eventual asteroid mining; and planetary defense strategies for potentially hazardous comets and asteroids. The main purposes of this paper are to relate the collisional disruption of objects such as comets and asteroids to their self-gravitational energy, and to demonstrate how to compute this self-energy.

The next section describes the collisional disruption of comets and asteroids, and the various terms in the impact energy budget, introducing gravitational "form factors" and an "energy rebate" for removal of only half their mass. Section 3 reviews the gravitational potential Φ and self-energy E_G of arbitrary mass distributions, spheres, ellipsoids, and polyhedra, and sets new analytic bounds on E_G. Section 4 describes our numerical methods for evaluating Φ, E_G, gravitational attraction, and surface slope for arbitrary homogeneous polyhedra. Section 5 applies these techniques to a homogeneous model of asteroid (216) Kleopatra, while Section 6 does the same for comet 67P/Churyumov–Gerasimenko. Section 7 generalizes our method to non-homogeneous polyhe-

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dra, and Section 8 summarizes our results. Finally, the Appendix tabulates the symbols used in this paper.

For the reader's convenience, the Supplementary Material includes explicit formulae for the second spacial derivatives of the gravitational potential (gravity gradient, or tidal tensor) of homogeneous polyhedra, and related formulae for the magnetic field and magnetic energy of uniformly magnetized polyhedra; provides detailed descriptions of the internal, external, and surface gravitational fields of homogeneous Platonic solids; and depicts the surface gravitational fields of homogeneous models for asteroid (216) Kleopatra and comet 67P/Churyumov–Gerasimenko.

In addition, the Supplementary Material reviews the gravitation of homogeneous ellipsoids and cuboids (rectangular parallelepipeds, or "box" shapes), simplifying certain classic results and deriving some new ones; and also plots the central potential, self-gravitational energy, and form factors for ellipsoids and cuboids of all aspect ratios. To date, these (or their degenerate cases, such as spheres or straight rods) are the only objects whose self-gravitational energy is known analytically (or semi-analytically). However, these are not always satisfactory models for the shapes of comets, asteroids, or satellites.

The Supplement also derives the self-gravitational energy of a "duplex" composed of two spheres; the gravitational field, moments of inertia, and various other properties of such duplexes were derived previously by Dobrovolskis and Korycansky (2013). Duplexes are more appropriate models for bilobate objects - those consisting of two parts stuck together in a "contact binary" resembling a peanut or a snowman. Many comets (such as 1P/Halley and 67P/Churyumov–Gerasimenko), asteroids (such as 216 Kleopatra and 25143 Itokawa), and possibly satellites (such as Pluto's small moon Kerberos; see Weaver and 50 co-authors (2016) are bilobate. In fact, 5 of the 7 well-resolved comets are bilobate (see Hirabayashi and 8 co-authors, 2016; the exceptions are 9P/Tempel 1 and 81P/Wild 2, which look more like potatoes). Thus most comets might be described better as "dirty snowmen" than as "dirty snowballs" (Whipple, 1950).

2. Collisional disruption

The *impact disruption energy* E_D of a comet or asteroid is defined as the minimum kinetic energy input needed both to shatter the object and to remove at least half of its mass (Davis *et al.*, 1977, see also Benz and Asphaug, 1999, and Asphaug et al., 2002). This disruption energy can be regarded as the sum of two terms; Formula (1) of Asphaug et al. (2002), derived ultimately from Fujiwara et al. (1977) by way of Davis et al. (1979), can be re-written as

$$E_D = E_S + E_B/e^*, \tag{1}$$

where E_S is the shattering energy of the object, and E_B is its binding energy. The dimensionless coefficient e^* may be regarded as an efficiency factor, because impacts transfer only a fraction e^* of their kinetic energy directly into the binding energy E_B of the target; note that $0 < e^* < 1$.

2.1. Shattering

The shattering energy E_S is the energy input needed just to shatter the object into many small pieces. In this context, the meaning of "many" and "small" depends on circumstances. For example, to shatter a contact binary composed of two identical spheres tangent from outside, it suffices to break the two lobes apart at their connection point. Many comets and asteroids may have no significant shattering energy, because they have been shattered already by non-disruptive impacts, or because they are accretional "rubble piles" of boulders and gravel with no significant cohesion.

Most asteroids smaller than $\sim$ 75 meters in radius rotate with spin periods of less than two hours, so they must have tensile strengths on the order of 0.02 MPa = 0.2 bars or more just to hold themselves together (Pravec et al., 2002); for comparison, the meteoroid which hit Chelyabinsk had a compressive strength on the order of 0.20 Mpa = 2 bars (Popova and 58 co-authors, 2013). Some of these rapidly spinning small asteroids may be solid, monolithic bodies; for these, the shattering energy E_S depends on the size, shape, strength, density, and porosity of the object.

Simple models suggest that the shattering energy E_S of a solid body should be proportional to its mass M (see Asphaug et al., 2002). Thus it is common to define an object's *specific* shattering energy $Q_S \equiv E_S/M$ as its shattering energy divided by its mass. Analogous definitions apply to its specific binding energy $Q_B \equiv E_B/M$, and to its specific disruption energy $Q_D \equiv E_D/M$. Thus formula (1) is equivalent to

$$Q_D = Q_S + Q_B/e^*. \tag{2}$$

Note that Q_D is usually called Q_D^* in the literature; and that the various specific energies $Q \equiv E/M$ all have dimensions of speed squared, and are not to be confused with the dimensionless "quality factor" Q conventionally used to parameterize damping of tides and of non-principal axis rotation.

Naïvely, the specific shattering energy Q_S should be independent of size, all else being equal. However, more sophisticated reasoning reveals that Q_S should decrease with increasing size of the object, based on the "weakest link" effect. As a measure of size, we define the volumetric radius (henceforth mean radius) $\bar{R}$ of an object as the radius of a sphere with the same mass M, volume V, and macroscopic density $\rho = M/V$ as the object:

$$\bar{R} \equiv \left(\frac{3V}{4\pi}\right)^{1/3} \approx 0.62035049\ (M/\rho)^{1/3}. \tag{3}$$

Then several arguments suggest that Q_S should scale roughly as the inverse square root of $\bar{R}$ (Asphaug et al., 2002).

2.2. Binding energy

An impact of energy E_S presumably shatters an object; but in order to disperse its mass, the object's gravitational binding energy E_B must be overcome as well. In this paper, we express E_B itself as the sum of three terms:[1]

$$E_B = E_G - E_2 - E_\omega, \tag{4}$$

where E_G is the self-gravitational energy (or just self-energy) of the object, which is the energy it would take to disperse its entire mass to infinity; E_2 is an energy "rebate", which we introduce as a correction term for breaking the body in two rather than into smithereens; and E_ω is the object's kinetic energy of rotation.

Analogously, we express the specific binding energy $Q_B \equiv E_B/M$ in terms of the specific self-energy $Q_G \equiv E_G/M$, the specific energy rebate $Q_2 \equiv E_2/M$, and the specific spin energy $Q_\omega \equiv E_\omega/M$:

$$Q_B = Q_G - Q_2 - Q_\omega. \tag{5}$$

[1] In order permanently to destroy a moon of mass M orbiting a planet of mass M_P at a semi-major axis s, the dispersed fragments must escape from the primary as well, lest they re-accrete into a second-generation satellite; see footnote 10 of Greenstreet et al. (2015). Then the binding energy contains a fourth term: $E_B = E_G - E_2 - E_\omega + E_O$, where $E_O = \frac{1}{2} G M_P M/s$ is the satellite's orbital energy. Likewise, Eq. (5) becomes $Q_B = Q_G - Q_2 - Q_\omega + Q_O$, where we define a moon's *specific* orbital energy $Q_O \equiv E_O/M = \frac{1}{2} G M_P/s$. Note that Q_O is independent of the satellite's mass M, its mean radius $\bar{R}$, and its other physical properties, while Q_G, Q_2, and Q_ω all scale as $M/\bar{R}$. For a homogeneous spherical moon of radius R, Q_O and E_O are actually greater than Q_G and E_G when $5R/s > 6M/M_P$.