Icarus 274 (2016) 83–98

Contents lists available at ScienceDirect

Icarus

journal homepage: www.elsevier.com/locate/icarus

Planetary and satellite three body mean motion resonances

Tabaré Gallardo*, Leonardo Coito, Luciana Badano

Departamento de Astronomía, Instituto de Física, Facultad de Ciencias, Universidad de la República, Iguá 4225, 11400 Montevideo, Uruguay

ARTICLE INFO

Article history:
Received 3 January 2016
Revised 17 March 2016
Accepted 20 March 2016
Available online 31 March 2016

Keywords:
Celestial mechanics
Planetary dynamics
Resonances
Orbital
Satellites
Dynamics

ABSTRACT

We propose a semianalytical method to compute the strengths on each of the three massive bodies participating in a three body mean motion resonance (3BR). Applying this method we explore the dependence of the strength on the masses, the orbital parameters and the order of the resonance and we compare with previous studies. We confirm that for low eccentricity low inclination orbits zero order resonances are the strongest ones; but for excited orbits higher order 3BRs become also dynamically relevant. By means of numerical integrations and the construction of dynamical maps we check some of the predictions of the method. We numerically explore the possibility of a planetary system to be trapped in a 3BR due to a migrating scenario. Our results suggest that capture in a chain of two body resonances is more probable than a capture in a pure 3BR. When a system is locked in a 3BR and one of the planets is forced to migrate the other two can react migrating in different directions. We exemplify studying the case of the Galilean satellites where we show the relevance of the different resonances acting on the three innermost satellites.

1. Introduction

One of the most prevalent dynamical phenomena observed in planetary systems is orbital commensurability, or resonance. Two body resonances (2BRs), extensively studied in orbital dynamics, occur when the ratio between the mean motions, n, of two bodies can be written as a fraction of 2 small integer numbers. They have proven to be very important in the architecture of the planetary systems (Batygin, 2015; Fabrycky et al., 2014). A less common case of resonance ensues when the mean motions of three bodies P_0, P_1 and P_2 verify

$$k_0 n_0 + k_1 n_1 + k_2 n_2 \simeq 0 \tag{1}$$

being k_i small integers, generating which is called a three body resonance (3BR). In some cases, the 3BRs can be the consequence of a chain of two 2BRs as is the case of the Galilean satellites studied since Laplace. In fact, the three innermost Galilean satellites, Io, Europa and Ganymede, verify the 2BR relations $n_I - 2n_E \sim 0$ and $n_E - 2n_G \sim 0$. Subtracting both expressions we obtain the 3BR $n_I - 3n_E + 2n_G \sim 0$, called Laplacian resonance. The resulting dynamics it is not a mere addition of the two 2BRs and the emerging 3BR generates a new complex dynamics. The Laplacian resonance is a paradigmatic case of a 3BR generated by the superposition or chains of two 2BRs. On the other hand, there are also 3BRs that cannot be decomposed as chains of 2BRs and we will call them *pure*. Thousands of asteroids in pure 3BRs with Jupiter and Saturn can be found in the Solar System (Smirnov and Shevchenko, 2013).

A relevant parameter of the 3BRs is the order defined as $q = |k_0 + k_1 + k_2|$. It is known that the lower the order the larger the dynamical effects of the resonance. That is why between the Galilean satellites the dominant 3BR is $n_I - 3n_E + 2n_G \sim 0$, and not for example $n_I - n_E - 2n_G \sim 0$ which is of order 2 and obtained adding the 2BRs instead of subtracting them. Note that the resonant condition (1) can be written as

$$k_1(n_1 - n_0) + k_2(n_2 - n_0) + (k_0 + k_1 + k_2)n_0 \simeq 0 \tag{2}$$

which means that for zero order resonances, even in the case of pure 3BRs, the planets P_1 and P_2 are in a simple 2BR k_1:k_2 when looked from the rotating frame of the planet P_0. No other 3BRs have this property which makes zero order 3BRs a special case. Then, it is not surprising that zero order 3BRs have been deserved most the attention. They were studied for example by Aksnes (1988) who obtained general formulae with applications in the asteroid belt and systems of satellites. The case of Laplacian resonance in the Galilean satellites has been intensely studied (Ferraz-Mello, 1979; Lainey et al., 2009; Malhotra, 1991; Peale and Lee, 2002; Showman and Malhotra, 1997; Showman et al., 1997; Sinclair, 1975). Superposition or chains of 2BRs were also studied in the major Saturnian satellites (Callegari and Yokoyama, 2010) and in extrasolar systems (Batygin et al., 2015; Batygin and Morbidelli, 2013; Libert and Tsiganis, 2011; Martí et al., 2013; Papaloizou, 2015). Quillen and French (2014) focused on systems with

* Corresponding author. Tel.: +598 25258618.
E-mail address: gallardo@fisica.edu.uy, tabarega@gmail.com (T. Gallardo).

http://dx.doi.org/10.1016/j.icarus.2016.03.018

close orbits with applications to the inner Uranian satellites, where it is remarked that 3BRs as consequence of superposition of first order 2BRs are the strongest ones. On the other hand, pure 3BRs were studied for example by Lazzaro et al. (1984) for the specific case of the Uranian satellites and by Nesvorný and Morbidelli (1999) where a complete planar theory was developed for the asteroidal, massless, case. The situation among the outer planets of the Solar System was analyzed numerically by Guzzo (2005); 2006). Quillen (2011) developed an analytical theory for general zero order resonances between three massive bodies in very close orbits while Gallardo (2014) developed a semianalytical method for estimation of the resonance's strength for pure 3BRs of any order for the asteroidal case assuming the perturbing planets in circular and coplanar orbits and the asteroid in an arbitrary orbit. Finally, it is worth mention that Showalter and Hamilton (2015) suggested that the satellites of Pluto, Styx, Nix and Hydra, are driven by the zero order 3BR $3n_S - 5n_N + 2n_H \sim 0$.

1.1. Looking for the disturbing function

The dynamics of a system trapped in a 3BR is determined by the resonant disturbing function, which its obtention is not a trivial point. The disturbing function for a 3BR emerges after a second averaging procedure applied on the resulting expressions of a first averaging involving the mutual perturbations between the planets taken by pairs (Nesvorný and Morbidelli, 1999). The final expression of the resonant disturbing function for planet P_0 assumed in the resonance given by Eq. (1) is a summatory of the type

$$\mathcal{R} = k^2 m_1 m_2 \sum_j \mathcal{P}_j \cos(\sigma_j) \tag{3}$$

where k is the Gaussian constant and m_1 and m_2 the planetary masses, with the critical angle

$$\sigma_j = k_0\lambda_0 + k_1\lambda_1 + k_2\lambda_2 + \gamma_j \tag{4}$$

and

$$\gamma_j = k_3\varpi_0 + k_4\varpi_1 + k_5\varpi_2 + k_6\Omega_0 + k_7\Omega_1 + k_8\Omega_2 \tag{5}$$

being λ, ϖ and Ω the mean longitudes, longitudes of the perihelia and longitudes of the nodes respectively, k_0, k_1, k_2 are integers fixed by the resonance and the $k_{i>2}$ are arbitrary integers but verifying the d'Alembert condition

$$\sum_{i=0}^{8} k_i = 0 \tag{6}$$

$\mathcal{P}_j$ is a polynomial function depending on the eccentricities and inclinations which its lowest order term is

$$C e_0^{|k_3|} e_1^{|k_4|} e_2^{|k_5|} \sin(i_0)^{|k_6|} \sin(i_1)^{|k_7|} \sin(i_2)^{|k_8|} \tag{7}$$

The calculation of the coefficients C is a very laborious task that must be done case by case and it is so challenging that only the planar case was studied by analytical methods and consequently there are not expansions involving $\sin(i_i)$ published up to now. An example of this development can be found in Gomes (2012) where an expansion for a specific 3BR in an extrasolar planar system is obtained. The expansion given by Eq. (3) implies that for a given resonance there are several σ_j contributing to the resonant motion. Each σ_j generates specific dynamical effects and the joint action of all σ_j is called multiplet. Nevertheless, the expansion (3) can be reduced to a few terms when the eccentricities and inclinations are very small. In particular, when $e_1 = e_2 = i_1 = i_2 = 0$ the lowest order non null terms for $\mathcal{P}_j$ are those with $k_4 = k_5 = k_7 = k_8 = 0$:

$$C e_0^{|k_3|} \sin(i_0)^{|k_6|} \cos(k_0\lambda_0 + k_1\lambda_1 + k_2\lambda_2 + k_3\varpi_0 + k_6\Omega_0) \tag{8}$$

from which can be deduced that for three coplanar orbits ($i_0 = 0$) the only non null terms are those with $k_6 = 0$, and consequently the lowest order term in the expansion is proportional to e_0^q, where $q = |k_3|$. This explain why the lower the order the stronger the resonance. In case that $e_0 = 0$ but with $i_0 \neq 0$ the non null terms are those with $k_3 = 0$ which result proportional to $\sin(i_0)^q$ instead, where $q = |k_6|$. But, as we explain below, if $|k_6|$ is odd the resulting principal term of the expansion is proportional to $\sin(i_0)^{2q}$. Note that for coplanar circular orbits all terms are null except for zero order resonances because in this special case the principal terms are independent of e_i, i_i.

To avoid the difficulties of the analytical methods Gallardo (2014) proposed a semianalytical method for the estimation of the strength of a resonance on a massless particle in an arbitrary orbit under the effect of two perturbing planets in circular coplanar orbits. The method, which is essentially an estimation of the amplitude of the disturbing function factorized by an arbitrary constant coefficient, was applied to minor bodies captured in 3BRs with the planets of the Solar System. In the present work, in Section 2 we extend the method to a system of three massive bodies with arbitrary orbits and we apply it to an hypothetical planetary system in order to analyze the dependence of the strengths on the orbital parameters. In Section 3 we explore by numerical methods some of the properties of the resonances that our method predicts and we apply the method to the case of the Galilean satellites. The conclusions are presented in Section 4.

2. Strength for planetary three body resonances and its dependence with the parameters

Strictly, 3BRs between three planets P_0, P_1 and P_2 with elements $(a_i, e_i, i_i, \Omega_i, \varpi_i)$ and masses m_0, m_1 and m_2 around a star of mass M occur when a particular critical angle given by Eq. (4) is oscillating over time. In this work we call $p = |k_0| + |k_1| + |k_2|$ and we note as $k_0 + k_1 + k_2$ the resonance involving the three planets, where always $k_0 > 0$. We will not consider the case of 3BRs as result of superposition of 2BRs because the 2BRs will override the dynamical effects of the 3BR we are trying to study, with the exception of systems with near zero eccentricity orbits. We will consider the planets P_1 and P_2 at fixed semimajor axes $a_1 < a_2$ and the third "test" planet P_0 with the semimajor axis defined by the resonant condition which can result in an internal, external o middle position with respect to P_1 and P_2. The approximate nominal location of the test planet P_0 assumed in the resonance $k_0 + k_1 + k_2$ is deduced from Eq. (1):

$$a_0^{-3/2} \simeq -\frac{k_1\sqrt{(M+m_1)}}{k_0\sqrt{(M+m_0)}} a_1^{-3/2} - \frac{k_2\sqrt{(M+m_2)}}{k_0\sqrt{(M+m_0)}} a_2^{-3/2} \tag{9}$$

which must be positive otherwise the resonance does not exist. In order to obtain a numerical estimation of the resonance's strength we extended the method given by Gallardo (2014) to a system of three massive bodies with arbitrary orbits. The details of the method and the devised algorithm can be found in the Appendix. Essentially, this new method predicts different strengths called S_0, S_1, S_2 for the three massive bodies, that means, each massive body feels the resonance in a different way. Each S is related to the amplitude of the variations of $\mathcal{R}$ in Eq. (3) caused by the cumulative effect of all involved terms. Then, the method cannot distinguish between the dynamical effects of each term of a multiplet for a given resonance, it only provides a global estimation.

In order to test the algorithm and to explore the dependence of the strengths with the different parameters involved we applied it to an hypothetical planetary system with $m_1 = m_2 = 0.0001 M_\odot$, $a_1 = 1.0$ au, $a_2 = 3.6$ au around a star with 1 $M_\odot$ and we calculate all resonances with $q \leq 9$ and $p \leq 30$ between 2.0 au and 2.6 au, that means with the planet P_0 located in between and excluding close-encounter situations. With the exception of Section 3.3,