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Tides of global ice-covered oceans

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ABSTRACT

The tides of an ice-covered ocean are examined using a Cartesian representation of the elastic and fluid equations. Although unconstrained by any observations, the ocean tides of a Neoproterozoic "snowball" Earth could have been significantly larger than they are today. Time-mean tidal-residual circulations would then have been set up that are competitive with the circulation driven by geothermal heating. In any realistic configuration, the snowball Earth would have had an ice cover that is in the thin shell limit, but by permitting the ice thickness to become large, more interesting ice tidal response can be found, ones conceivably of application to bodies in the outer Solar System or hypothetical exoplanets. Little can be said concerning a reduction in tidal dissipation necessary to avoid a crisis in the history of the lunar orbit.

1. Introduction

Several reasons exist for an exploration of the tides occurring in and under ice sheets, whether floating or land-confined. One motive arises from evidence that approximately 600 million years ago, during the Neoproterozoic, the entire Earth may have frozen, being everywhere covered with ice. Over the ocean, a floating ice sheet may have existed with an estimated thickness of several kilometers (the "hard snowball Earth"). Discussion of the evidence, primarily geological in nature, can be found in Hoffman and Schrag (2002). Ashkenazy et al. (2014, hereafter, A14), describe a theoretical/modeling study of the oceanic circulation that might exist under an oceanic ice cover of order of several kilometers. The forcing they assume is purely geothermal, at the average modern rate of roughly 0.1 W/m^2 (Davies, 2013; Pollack et al., 1993), with some localized maxima over ridge-crests. They find an equatorially enhanced meridional overturning circulation, with transports up to 30 $\times$ 10^{6}m^3/s (30 Sverdrups; Sv) with a nearly homogeneous ocean, both in temperature and salinity. Some account is taken of the oceanic interaction with the overlying ice sheet. Jansen (2016) has in turn suggested that the resulting flow would be a turbulent one.

Whether or not a complete snowball Earth actually existed, the question of what the ocean might be like under such circumstances is an interesting theoretical problem. A modern analogue may lie in the outer Solar System satellites Enceladus and Europa, which have been inferred to contain fluid oceans covered by multi-kilometer thick ice sheets. In contrast to the A14 solution, discussion of behavior of those oceans has centered on tidal forcing (e.g., Beuthe, 2015; Greenberg, 1998; Tyler, 2008; Vance and Goodman, 2009).

Another motivation arises from the known difficulties in accounting for the history of the lunar orbit. The existing rate of tidal dissipation, if constant through time, would have brought the Moon catastrophically close to the Earth about 1 billion years ago (e.g., Goldreich, 1966; Macdonald, 1964; Munk, 1968). Munk called the catastrophe the "Gerstenkorn event," and which is known not to have occurred. The conventional interpretation is that lunar tidal dissipation must have been greatly reduced some hundreds of millions of years in the past (see Bills and Ray, 1999 for discussion). Should tidal dissipation have been much reduced during the approximately 200MY of the Neoproterozoic, it would be a significant contribution to explaining how the reduction occurred.[1]

A comparatively large literature exists on tides induced in ice sheets by the oceanic tidal forcing at the outflow (e.g., Arbic et al.,

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[1] Bartlett and Stevenson (2015) have revived a suggestion of Holmberg (1952), that the principal atmospheric solar semi-diurnal tide—which today effectively accelerates the Earth's rotation—was in a resonant steady-state through much of Earth's history, terminating with the end of the Neoproterozoic. See Munk and MacDonald (1960) for a review of the atmospheric resonance theory, which dates back to Rayleigh's work. Discussion of whether such a high degree of resonance existed and whether it could have persisted through order 1 GY of Earth history is far beyond the scope of the present paper.

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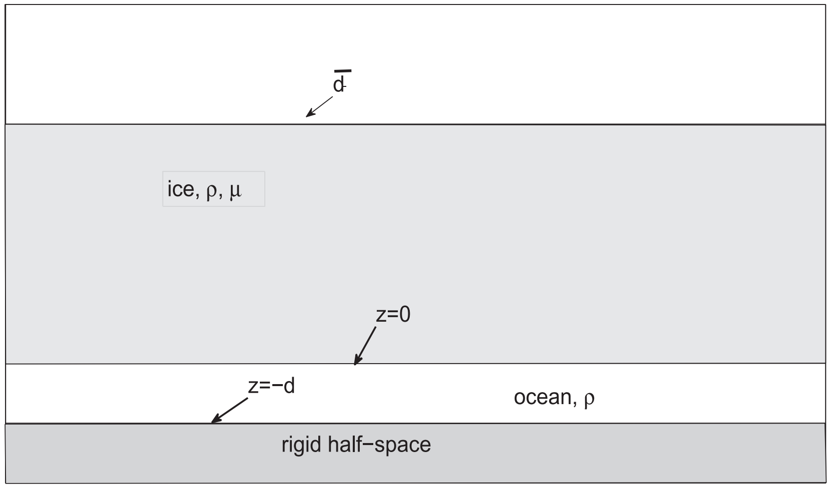

Fig. 1. Defining geometry of an ice layer of thickness $\bar{d}$ over an ocean of depth d. Below the ocean is an infinite, rigid, half-space. z is directed vertically upward from the ice–ocean interface.

2008; Reeh et al., 2003; Thomas, 2007). These effects are of at least tangential interest here, but where the focus is instead on the directly driven tidal motions within the ice. Some of the parameter ranges used here are far beyond anything reasonable for the Earth. Perhaps they have some relevance for another planet or satellite.

2. A Cartesian configuration

Because of all of the uncertainties of the physical setting of the Neoproterozoic Earth, the restricted goals here are to understand the basic physics and to find orders of magnitude of the effects. Only a two-dimensional Cartesian system, as in the Airy "canal theory" of water tides (Lamb, 1932), is used. Consider the situation in Fig. 1, in which an ice sheet of uniform thickness $\bar{d}$ overlies an ocean of constant depth d; on the Earth, $d, \bar{d}$ would have an inverse relationship over time. Below the ocean is an infinite elastic half-space. The fluid motion is computed with the half-space not moving, and the ocean tide is computed relative to the sea floor. Conceptually, as with ocean tides measured from tide gauges, tides within the elastic half-space will produce a modified tidal potential, $U = U_0(1 + k_L - h_L)$, where U_0 is the gravitational disturbing potential and k_L, h_L are the conventional Love numbers (Lambeck, 1988; Munk and MacDonald, 1960). The net tide generating potential will be assumed to be,

$$U = gHe^{ikx-i\sigma t} = g\eta_{Eq}, \tag{1}$$

so that the fluid equilibrium height would be $|\eta_{Eq}| = H$, but with the half-space treated here as completely rigid (unmoving).

2.1. Equations of an elastic sheet

Rheological properties of ice, whether on land or floating, are not simple—encompassing elastic, viscous, plastic and fracture flow laws. MacAyeal and Sergienko (2013) proposed that for time-scales of less than about 10 days, treating sea ice as elastic is appropriate and thus reasonable for describing ordinary semi-diurnal or diurnal tides. (Reeh et al., 2003, conclude that incorporation of a land-based part of the glacier would require a viscoelastic rheology, but this bending-beam limit is not undertaken here.) Discussion of long-period tides, including the orbital- rather than rotation-dominated ones in outer-Solar System satellites, requires revisiting the rheology and invoking all of viscoelastic, plastic, and fracture mechanisms.

The ice is treated here as purely elastic with Lamé constants λ, μ and the physical and mathematical structure of the problem corresponds to the free-mode analyses of Bromwich (1898); Greenhill (1886); Press and Ewing (1951), and Ewing et al. (1957, chap. 5), but in the presence of a periodic body-force of radian frequency σ. The Cartesian system governing an elastic plate is

$$\bar{\rho}\frac{\partial^2 \bar{u}}{\partial t^2} = -\sigma^2\bar{\rho}\bar{u} = (\lambda+\mu)\frac{\partial}{\partial x}\left(\frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{w}}{\partial z}\right) + \mu\nabla^2\bar{u} + \bar{\rho}\frac{\partial U}{\partial x} \tag{2a}$$

$$\bar{\rho}\frac{\partial^2 \bar{w}}{\partial t^2} = -\sigma^2\bar{\rho}\bar{w} = (\lambda+\mu)\frac{\partial}{\partial z}\left(\frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{w}}{\partial z}\right) + \mu\nabla^2\bar{w} - g\bar{\rho} \tag{2b}$$

Variables $\bar{u}, \bar{w}$ in the plate are *displacements,*not velocities. Barred variables will refer to displacements in the ice layer, unbarred ones to corresponding *velocities* in the ocean. $\bar{\rho}$ is the density of ice, U has no vertical dependence in the ocean or ice layers and no $y-$dependence is considered. The background gravity g produces a resting static pressure $\bar{\rho}g(\bar{d} - z)$ in the ice; the resulting compaction and induced gravity disturbance are neglected here.

What follows is in the spirit of the paper by Bromwich (1898) and who, as in the papers of Rayleigh and Love, defined the pressure as,

$$\bar{p}(x,z) = -\lambda\left(\frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{w}}{\partial z}\right), \tag{3}$$

taken as finite, but otherwise treated the medium as incompressible with,

$$\frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{w}}{\partial z} = 0, \tag{4}$$

and implying $\lambda \to \infty$. One advantage of this system is that it increases the resemblance between the elastic and fluid equations. The sign of $\bar{p}$ has been reversed here from the Bromwich definition, conventional in elasticity, in the interests of that analogy.

In any realistically ice-covered Earth-like ocean, the ice sheet thickness would be a very small fraction of the tidal wavelength, suggesting the use of equilibrium thin-plate theory (e.g., Greenhill, 1886; Landau and Lifshits, 1970; Turcotte and Schubert, 2002) instead of the dynamical wave equations. That course is not followed so as to make it possible to include the interesting situation in which much thicker ice sheets are disturbed by tides, a configuration perhaps existing in theory in the outer Solar System or among exoplanets. The "thin-shell" fluid ocean, which gives rise to the Laplace Tidal Equations used here (Cartesian limit), is likely inappropriate for outer Solar System satellites, for which a non-hydrostatic, fully spherical coordinate system would be required—as in the Earth's core (Melchior, 1983, chap. 6; Harrison, 1985).