Icarus 277 (2016) 416–423

Contents lists available at ScienceDirect

Icarus

journal homepage: www.elsevier.com/locate/icarus

Odd gravitational harmonics of Jupiter: Effects of spherical versus nonspherical geometry and mathematical smoothing of the equatorially antisymmetric zonal winds across the equatorial plane

Dali Kong[a,b], Keke Zhang[b,c,*], Gerald Schubert[d]

[a] Key Laboratory of Planetary Sciences, Shanghai Astronomical Observatory, Chinese Academy of Sciences, Shanghai 200030, China
[b] Center for Geophysical and Astrophysical Fluid Dynamic, College of Engineering, Mathematics and Physical Sciences, University of Exeter, Exeter EX4 4QF, UK
[c] Lunar and Planetary Science Laboratory, Macau University of Science and Technology, Macau
[d] Department of Earth, Planetary and Space Sciences, University of California, Los Angeles, CA 90095-1567, USA

ARTICLE INFO

Article history:
Received 14 July 2015
Revised 14 May 2016
Accepted 26 May 2016
Available online 2 June 2016

Keywords:
Jupiter
Jupiter's interior
Jupiter's atmosphere

ABSTRACT

Unlike the even gravitational coefficients of Jupiter that are caused by both the rotational distortion and the equatorially symmetric zonal winds, the odd jovian gravitational coefficients are directly linked to the depth of the equatorially antisymmetric zonal winds. Accurate estimation of the wind-induced odd coefficients and comparison with measurements of those coefficients would be key to understanding the structure of the zonal winds in the deep interior of Jupiter. We consider two problems in connection with the jovian odd gravitational coefficients. In the first problem, we show, by solving the governing equations for the northern hemisphere of Jupiter subject to an appropriate condition at the equatorial plane, that the effect of non-spherical geometry makes an insignificant contribution to the lowermost-order odd gravitational coefficients. In the second problem, we investigate the effect of the equatorial smoothing used to avoid the discontinuity in the winds across the equatorial plane when the thermal wind equation is adopted to compute the odd gravitational coefficients. We reveal that, because of the dominant effect of the equatorial smoothing, the odd gravitational coefficients so obtained for deep zonal winds do not reflect physically realistic dynamics taking place in the deep interior of Jupiter.

1. Introduction

The zonal external potential V_g of the jovian gravitational field can be expanded in terms of the Legendre functions P_n,

$$V_g = -\frac{GM}{r}\left[1 - \sum_{n=2}^{\infty} J_n \left(\frac{R_e}{r}\right)^n P_n(\cos\theta)\right], \quad r \geq R_e, \tag{1}$$

where M is Jupiter's mass, n takes integer values, $J_2, J_3, J_4, J_5, \ldots,$ are the zonal gravitational coefficients, (r, θ, ϕ) are spherical polar coordinates with the corresponding unit vectors $(\hat{\mathbf{r}}, \hat{\boldsymbol{\theta}}, \hat{\boldsymbol{\phi}})$ and $\theta = 0$ is at the axis of rotation, R_e is the equatorial radius of Jupiter, and G is the universal gravitational constant ($G = 6.67384 \times 10^{-11}\,\mathrm{m^3 kg^{-1} s^{-2}}$). Jupiter is rotating rapidly and its shape significantly departs from spherical geometry: the eccentricity at the one-bar surface is $\mathcal{E}_J = 0.3543$ (Seidelmann et al., 2007). Both the rotational distortion and the equatorially symmetric zonal winds, if sufficiently deep, contribute to the even gravitational coefficients J_n with $n \geq 2$ in (1). Gravity measurements by the Juno spacecraft provide only the total gravitational coefficients (Bolton, 2005) and accurately identifying the wind-induced contribution from the measured values J_n represents a difficult task. By contrast, the rotational distortion, because of its equatorial symmetry, does not contribute to the odd gravitational coefficients J_n with $n \geq 3$ in (1). Hence, the odd gravitational coefficients are determined only by the equatorially antisymmetric zonal winds. These coefficients will be detectable by the high-precision gravitational measurements of the Juno spacecraft.

There exist two studies concerned with estimating the odd gravitational coefficients of Jupiter from its equatorially antisymmetric zonal winds. Using the thermal wind equation in spherical geometry, Kaspi (2013) calculated the gravitational signature induced by the equatorially antisymmetric zonal winds $U(r, \theta)$ in

* Corresponding author at: Center for Geophysical and Astrophysical Fluid Dynamic, College of Engineering, Mathematics and Physical Sciences, University of Exeter, Exeter EX4 4QF, UK. Tel.: +44 (0) 1392 725226.
E-mail addresses: D.Kong@exeter.ac.uk (D. Kong), kzhang@ex.ac.uk, K.Zhang@exeter.ac.uk (K. Zhang), schubert@ucla.edu (G. Schubert).

http://dx.doi.org/10.1016/j.icarus.2016.05.037
0019-1035/

the form

$$U(r,\theta) = +u_0(r\sin\theta)\mathrm{e}^{-(R_S-r)/H}, \quad 0 \le \theta \le \pi/2;$$
$$U(r,\theta) = -U(r,\pi-\theta), \quad \pi/2 < \theta \le \pi, \tag{2}$$

where R_S is the radius of the planet, $s = r\sin\theta$ denotes the distance from the rotation axis, $u_0(r\sin\theta)$ represents the equatorially antisymmetric component of the observed cloud-level zonal winds in the northern hemisphere extending into the equator on cylinders parallel to the rotation axis and H is a depth parameter. Three important features should be highlighted: (i) $U(r, \theta)$ given by (2) is discontinuous across the equatorial plane, (ii) both $U(r, \theta)$ and its derivative $\partial U/\partial\theta$ are mathematically undefined at $\theta = \pi/2$, and (iii) since $U(r, \theta)$ is equatorially antisymmetric, it suffices to consider either the southern or northern hemisphere of the whole sphere. Kong et al. (2015b) computed the odd gravitational coefficients induced by the deep equatorially antisymmetric zonal winds in an oblate spheroidal geometry (Kong et al., 2015a) by imposing the equatorially antisymmetric condition at the equatorial plane and, then, solving the governing equations in the northern hemisphere. Note that the hemispheric model (Kong et al., 2015b) has to assume that the antisymmetric wind velocities do not vary along z, the coordinate parallel to the rotation axis, in the non-equatorial regions while the thermal-wind-equation approach (Kaspi, 2013) does not need to make the z-independent assumption. Differences between the results of Kong et al. (2015b) and Kaspi (2013) are large. For example, Kaspi (2013) obtained $J_7 \approx 6 \times 10^{-7}$ for an asymptotically large H, while Kong et al. (2015b) found that $J_7 = -7.4 \times 10^{-7}$; there is also an O(100)% difference in the size of J_3. Identifying the origin of such large differences is important because the odd gravitational coefficients will play a key role in interpreting the high-precision gravitational measurements made by the Juno spacecraft. There are shortcomings of both the hemispheric barotropic model (Kong et al., 2015b) and the equatorial smoothing model (Kaspi, 2013) that will be discussed later.

This study examines two possible effects that might be responsible for causing such large differences. The first is the effect of geometry—spherical versus non-spherical—that would cause some differences in the values of odd gravitational coefficients. We repeat the non-spherical computation performed by Kong et al. (2015b) but with spherical geometry. It turns out that the effect of non-spherical geometry contributes less than 10% to the lowermost-order odd gravitational coefficients. The second effect is more subtle and is concerned with how to smooth the equatorially antisymmetric zonal winds which are assumed to be discontinuous across the equatorial plane (Kaspi, 2013). When using the thermal-wind-equation approach, one has to compute $\partial U/\partial\theta$ near the equatorial plane where the derivative is discontinuous. In order to avoid the discontinuity, an equatorial smoothing that connects the northern profile [$U(r, \theta)$ for $0 \le \theta < \pi/2$] with the southern profile [$U(r, \theta)$ for $\pi/2 < \theta \le \pi$] in the equatorial region may be adopted such that $\partial U/\partial\theta$ can be conveniently evaluated there. By adopting the Gaussian smoothing function (for example, Lin et al., 1999) in the equatorial region, we are able to reproduce the results of Kaspi (2013) for the deep equatorially antisymmetric zonal winds. We show that the effect of equatorial smoothing—which results from an infinitely large shear assumed at the equatorial plane that, due to the action of jovian interior magnetic fields and viscosity, is physically and dynamically infeasible—is overwhelmingly dominant and, consequently, the odd gravitational coefficients produced chiefly by equatorial smoothing have no physical significance.

We begin in Section 2 by solving the governing equations in the northern hemisphere of Jupiter subject to an appropriate condition at the equatorial plane in spherical geometry in order to understand the geometric effect on the odd gravitational coefficients. The subtle but critically important effect of the equatorial smoothing on the odd gravitational coefficients is discussed in Section 3 with conclusions and some remarks given in Section 4.

2. Odd J_n based on hemispheric computation

Our model assumes that (i) Jupiter with mass M and the radius R_S is isolated and rotating about the symmetry z-axis with an angular velocity $\Omega\hat{\mathbf{z}}$; (ii) the effect of the rotational distortion on estimating the odd gravitational coefficients can be neglected; (iii) Jupiter is axially symmetric and consists of a compressible barotropic fluid (a polytrope of index unity) whose density ρ is a function only of the pressure p (Hubbard, 1999), and (iv) the zonal winds observed on Jupiter have an equatorially antisymmetric component that depends only on distance s from the rotation axis and extends from the cloud level to the equatorial plane, which represents the large H limit in the profile (2). In an inertial frame of reference, the equilibrium equations are

$$\mathbf{u}\cdot\nabla\mathbf{u} = -\frac{1}{\rho}\nabla p - \nabla V_g, \tag{3}$$

$$p = K\rho^2, \tag{4}$$

$$\nabla^2 V_g = 4\pi G\rho, \tag{5}$$

$$\nabla\cdot(\mathbf{u}\rho) = 0, \tag{6}$$

where K is a constant, $\mathbf{u}$ denotes the fluid motion and V_g represents the gravitational potential. Eqs. (3)–(6) are solved subject to the boundary condition

$$p(r,\theta) = \rho(r,\theta) = 0 \quad \text{at} \quad r = R_s, \tag{7}$$

where the effect of the rotational distortion is neglected. Since the wind speed is much smaller than that of the solid-body rotation, we can solve Eqs. (3)–(6) by making the expansions

$$\rho = \rho_0 + \rho_1, \quad p = p_0 + p_1, \quad \mathbf{u} = \Omega\hat{\mathbf{z}}\times\mathbf{r} + \widetilde{U}(r,\theta)\hat{\boldsymbol{\phi}}, \tag{8}$$

where $\widetilde{U}(r,\theta)\hat{\boldsymbol{\phi}}$, displayed in Fig. 1, represents the observed cloud-level, equatorially antisymmetric zonal winds (Porco et al., 2003) extending into the equatorial plane on cylinders parallel to the rotation axis.

While the leading-order problem for ρ_0 and p_0 can be solved analytically in spherical geometry, our focus is on computing the lowermost odd zonal gravitational coefficients J_3, J_5, J_7 in the expansion (1) induced by the deep equatorially antisymmetric winds $\widetilde{U}(r,\theta)$ satisfying

$$\widetilde{U}(r,\theta) = -\widetilde{U}(r,\pi-\theta) \quad \text{for} \quad 0 \le \theta < \pi/2. \tag{9}$$

The equatorially antisymmetric winds only drive the density anomaly ρ_1 obeying the parity

$$\rho_1(r,\theta) = -\rho_1(r,\pi-\theta),$$

which only produces the odd gravitational coefficients J_n. In addition to the boundary condition at the spherical surface

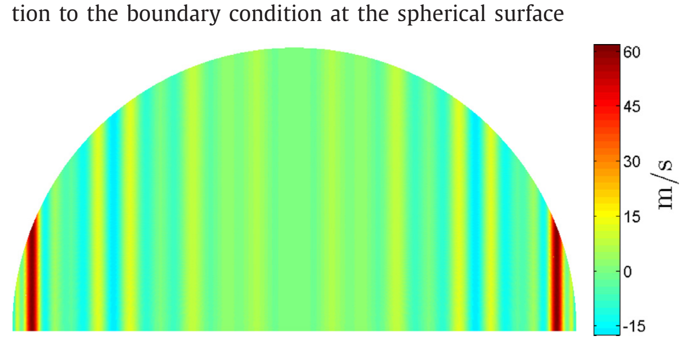

Fig. 1. The equatorially anti-symmetric jovian zonal winds $\widetilde{U}(r,\theta)$ in the northern hemisphere obtained by extending the observed cloud-level zonal winds (Porco et al., 2003) into the interior on cylinders parallel to the rotation axis. The antisymmetric zonal winds in the southern hemisphere are given by $-\widetilde{U}(r,\pi-\theta)$.