Icarus 266 (2016) 173–188

Contents lists available at ScienceDirect

Icarus

journal homepage: www.journals.elsevier.com/icarus

A fast method for quantifying observational selection effects in asteroid surveys

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ARTICLE INFO

Article history:
Received 17 December 2013
Revised 13 September 2015
Accepted 26 October 2015
Available online 31 October 2015

Keywords:
Near-Earth Objects
Asteroids, dynamics
Data reduction techniques
Asteroids

ABSTRACT

We present a fast method to calculate an asteroid survey's 'bias' – essentially a correction factor from the observed number of objects to the actual number in the population. The method builds upon the work of Jedicke and Metcalfe (Jedicke, R., Metcalfe, T.S. [1998]. Icaurs 131, 245–260) and Granvik et al. (Granvik, M., Vaubaillon, J., Jedicke, R. [2012]. Icarus 218, 262–277) and essentially efficiently maps out the phase space of orbit elements that can appear in a field-of-view. It does so by 'integrating' outwards in geocentric distance along a field's boresite from the topocentric location of the survey and calculating the allowable angular elements for each desired combination of semi-major axis, eccentricity and inclination. We then use a contour algorithm to map out the orbit elements that place an object at the edge of the field-of-view. We illustrate the method's application to calculate the bias correction for near Earth Objects detected with the Catalina Sky Survey (Christensen, E. et al. [2012]. AAS/Division for Planetary Sciences Meeting Abstracts, vol. 44, p. 210.13; Larson, S. et al. [1998]. Bulletin of the American Astronomical Society, vol. 30, p. 1037).

1. Introduction

The critical key to measuring the true size and orbit distribution of an asteroid or comet population is the efficiency for detecting them *as a function of their size and orbit elements*, and the fidelity of the detection efficiency needs to improve in step with the number of known objects in the population in order that the corrected, unbiased, population estimate is not dominated by systematic errors and uncertainties. In this work we describe a faster method for calculating asteroid detection efficiency (also known as the 'bias correction') for modern high-statistics long-duration asteroid surveys that builds upon the methods described by Jedicke and Metcalfe (1998) and Jedicke et al. (2002).

There is a long history of attempting to determine the true number of asteroids in the solar system. For instance, Baade (1934) estimated that there are 30–40 thousand asteroids brighter than $V = 19$ on the sky using just 37 asteroids detected on 21 photographic plates. The methods he employed to correct for the loss in detection sensitivity as a function of the asteroid's trail length on the exposure were similar to those employed today with CCDs. His bias correction from the area covered by the 21 photographic plates to the entire sky relied on the 1200 asteroids known at the time but did not account for the difference in distance between main belt objects observed towards opposition and those in the direction of the Sun. Baade's (1934) estimate was $\gtrsim 10\times$ Jehkowsky's[1] estimate from just a year earlier but is in fairly good agreement with modern estimates – there are about 25,000 known asteroids with $V < 19$ more than 90° from the Sun, and the Solar System model of Grav et al. (2011) suggests that there are about 40,000 asteroids with $V < 19$ on the entire sky-plane.

The all-ecliptic McDonald Survey of Asteroids to $B \lesssim 16.5$ (Kuiper et al., 1958) and its extension, the 'deep' pencil-beam Palomar–Leiden Survey to $B \lesssim 20$ (PLS, van Houten et al., 1970), 'debiased' their results but it is difficult to compare the modern population to their values because of the different magnitude systems and their non-physical limitations on the population, e.g. restricting the population of objects to those with declination <18°. If we assume that the PLS absolute g magnitudes were photographic B and use a mean $B - V$ asteroid color of 0.75 (Tedesco,

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[1] From Baade (1934): Jehkowsky, *Sur le nombre probable d'asterodes que l'on peut découvrir avec les moyens actuels d'observation.* C.R., 197, 579, 1933.

http://dx.doi.org/10.1016/j.icarus.2015.10.021

1995) then the PLS estimated that there are about 18,000 asteroids with absolute magnitude $H_V < 15.0$ in the declination strip. This is about half the number of *known* asteroids with $H_V < 15.0$ and inclinations $i < 9^\circ$, corresponding roughly to those asteroids that are confined to ecliptic latitudes of $\beta \lesssim 18^\circ$.

About 40 years ago Whipple (1973) stated that the frequency distribution of Apollo-type asteroids, those with perihelion distance < 1 au, 'remains quite uncertain'. He attempted to correct for the selection effects by using the fact that none of the 15 Apollo asteroids known at the time had been accidentally re-discovered. This allowed him to state with 50% confidence that the total number of Apollo asteroids with $H < 18$, roughly 1 km diameter or larger, must be less than 100. The known number[2] of Apollos in this size range is currently about 6× larger.

The lesson from these early attempts to debias the observed population is that correcting for observational selection effects is not easy. It is not appropriate to make simple single-parameter (e.g. absolute magnitude) corrections because there is a complicated interplay between the parameters that determine whether an asteroid will be detected by a survey and the underlying orbit element and size–frequency distribution.

Spahr (1998) and Jedicke and Metcalfe (1998) independently and nearly at the same time implemented the first 'modern' attempts to simultaneously compensate for main belt asteroid survey selection effects in the 4-dimensions of semi-major axis, eccentricity, inclination and absolute magnitude. The former work re-calculated the observational biases for the PLS and also applied their method to a new survey optimized for detecting high-inclination objects. The latter work was used to determine the biases in the Spacewatch near-Earth object (NEO) survey (e.g. Larsen et al., 2001) and to fit the observed NEO distribution to a theoretical NEO model (Bottke et al., 2002). They predicted that there were 960 ± 120 NEOs with $H < 18$, about 2-σ below the almost 1200 NEOs now known in that size range. The NEO population is thought to be $\gtrsim 90\%$ complete so it is unlikely that the (Bottke et al., 2002) result will turn out be in error by more than a few sigma. The 40% error in the Bottke et al. (2002) NEO model is more than 10× less than the ~600% error in Whipple's (1973) prediction from 30 years earlier and it is desirable to reduce the error in future models that extend the size distribution to smaller asteroid sizes.

There are two main problems in calculating survey biases for asteroids. First, asteroid surveys have focussed more on ambitiously discovering new objects at the expense of rigorously quantifying their detection capability as a function of apparent magnitude and rate of motion. These are the two main observables that determine whether an asteroid is discovered and they depend directly on the object's orbit and physical characteristics and are required for an accurate determination of the bias. Second, it is computationally expensive to calculate a survey's asteroid detection efficiency (bias correction). The calculation is even more complicated for comets because of their variable phase functions and the problems associated with detecting 'fuzzy' image features or those with tails and even more complicated morphologies.

In this work we mostly address the computational issue by introducing a method that efficiently determines the orbit element phase space of objects that can appear in a field, thereby eliminating the need to calculate ephemerides for objects that never do. We apply the method to the Catalina Sky Survey for NEOs (Christensen et al., 2012; Larson et al., 1998) because they have measured their detection efficiency on a nightly basis and characterized it as a function of a detection's trail length.

[2] We ignore the ~60 sub-components of the disrupted comet 73P/Schwassmann–Wachmann 3.

2. Survey bias

In this section we define and derive our calculation of an asteroid survey 'bias' that is a correction factor from the observed number of asteroids to the actual number in a desired sub-population.

Let $\vec{x} \equiv (a, e, i)$ represent an asteroid orbit's semi-major axis a, eccentricity e, and inclination i, while $\vec{y} \equiv (\Omega, \omega, M)$ represents the orbit's angular elements; the longitude of ascending node Ω, the argument of perihelion ω, and mean anomaly M. Let H represent the asteroid's absolute magnitude.[3] Furthermore, let $\vec{z} \equiv (\vec{x}, \vec{y}, H) = (a, e, i, \Omega, \omega, M, H)$ represent the set of six elements and H. Then the detected number of objects in a single field of view (FOV) j in an infinitesimal range $[\vec{z} - d\vec{z}/2, \vec{z} + d\vec{z}/2]$ is

$$n_j(\vec{z})\, d\vec{z} = \epsilon_j(\vec{z})\, N(\vec{z})\, d\vec{z} \tag{1}$$

where $d\vec{z} \equiv (da, de, di, d\Omega, d\omega, dM, dH)$, $N(\vec{z})\, d\vec{z}$ is the actual number of objects in the same range of $\vec{z}$, $\epsilon_j(\vec{z})$ is the efficiency for detecting objects with $\vec{z}$ in the field, and $N(\vec{z})$ and $n_j(\vec{z})$ are the actual and observed number densities at $\vec{z}$.

The actual distribution of objects $N(\vec{z})$ can only be determined with a good measurement of the efficiency and a large number of detected objects. Since the number of objects in any single FOV is small, and the efficiency of detecting them is explicitly a function of their apparent brightness and rate of motion and only implicitly dependent on the orbit elements and absolute magnitude, the calculation of $N(\vec{z})$ typically requires a large number of fields j and an accurate measurement of the detection efficiency. Thus, the total number of detected objects n' in an infinitesimal range $[\vec{z} - d\vec{z}/2,\ \vec{z} + d\vec{z}/2]$ detected during a survey with many fields of view is

$$n'(\vec{z})\, d\vec{z} = \sum_j \epsilon_j(\vec{z})\, N(\vec{z})\, d\vec{z} \equiv b(\vec{z})\, N(\vec{z})\, d\vec{z}. \tag{2}$$

In this formulation we allow that the same object may be detected multiple times in different fields and we introduce $b(\vec{z})$, a correction factor from the number of detected objects with $\vec{z}$ in all the fields to the actual number of objects with $\vec{z}$ in the population.

At the current time there are about 500,000 known asteroids so that if the orbits were randomly distributed, and there were just ten bins in each dimension of a 6-dimensional orbit element space $(a, e, i, \Omega, \omega, M)$, there would be only about 0 or 1 entry in each bin, i.e. there is not much resolution in each dimension even with $\mathcal{O}(10^6)$ orbits. To increase the resolution in each bin we integrate over a limited range of the dimensions so that the detected number of objects in the range $[\vec{z}_1, \vec{z}_2]$ is

$$n'(\vec{z}_1, \vec{z}_2)\, \Delta\vec{z} = \int_{\vec{z}_1}^{\vec{z}_2} b(\vec{z})\, N(\vec{z})\, d\vec{z} \tag{3}$$

where $\Delta\vec{z} \equiv \vec{z}_2 - \vec{z}_1$.

If $\Delta\vec{z}$ is small and b a slowly varying function of $\vec{z}$ we can make the approximation that

$$n'(\vec{z}_1, \vec{z}_2)\, \Delta\vec{z} = B(\vec{z}_1, \vec{z}_2)\, N(\vec{z}_1, \vec{z}_2)\, \Delta\vec{z} \tag{4}$$

where we refer to $B(\vec{z}_1, \vec{z}_2)$ as the survey 'bias', the 'correction factor' from the number of detected objects to the actual number of objects $N(\vec{z}_1, \vec{z}_2)$ in the range $[\vec{z}_1, \vec{z}_2]$. Thus,

$$B(\vec{z}_1, \vec{z}_2) = \frac{\int_{\vec{z}_1}^{\vec{z}_2} b(\vec{z})\, N(\vec{z})\, d\vec{z}}{N(\vec{z}_1, \vec{z}_2)\, \Delta\vec{z}} \equiv \sum_j \hat{\epsilon}_j(\vec{z}_1, \vec{z}_2) \tag{5}$$

from which it is clear that the bias is the sum over all the survey fields of the number-weighted average efficiency, $\hat{\epsilon}_j(\vec{z}_1, \vec{z}_2)$, for

[3] We use the IAU standard H-G_{12} system Muinonen et al. (2010) with $G_{12} = 0.5$.